Chiral Perturbation Theory in Kaon Decay

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Radiative

- Introduction
- $K \to \pi\pi\gamma$ at $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$
- Results from KTeV and BNL-E787
- Discussions

Chiral Symmetry

QCD lagrangian is invariant under

$$SU(3)_V \times SU(3)_A = SU(3)_L \times SU(3)_R .$$

$$\begin{bmatrix} u_L' \\ d_L' \\ s_L' \end{bmatrix} = R \begin{bmatrix} u_L \\ d_L \\ s_L \end{bmatrix} , \quad \begin{bmatrix} u_R' \\ d_R' \\ s_R' \end{bmatrix} = R' \begin{bmatrix} u_R \\ d_R \\ s_R \end{bmatrix}$$

mass term $\quad q_L \leftrightarrow q_R \quad \Rightarrow \cancel{SU(3)_A}$

If $SU(3)_A$ is Spontaneously broken, there must be 8 Nambu-Goldstone bosons.

$$= \pi , K , \eta$$

The Lowest Order Chiral ChPT Lagrangian

$$U = \exp\left(i\sqrt{2}\Phi/F\right), \qquad \Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}$$

$$\mathcal{L}_2 = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \rangle$$

$$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu\,, \qquad \chi = 2B(s + ip)$$

$U \to R U L^\dagger$

$D_\mu U D^\mu U^\dagger \to R D_\mu U L^\dagger L D^\mu U^\dagger R^\dagger$

$$M^2_{\pi^+} = B(m_u + m_d) + O(m^2_{quark})$$
$$M^2_{K^+} = B(m_u + m_s) + O(m^2_{quark})$$

Weak Decays

$$\mathcal{L}_W = G_8 F^4 (L_\mu L^\mu)_{23} + G_{27} F^4 \left[(L_\mu)_{23} (L^\mu)_{11} + \frac{2}{3} (L_\mu)_{21} (L^\mu)_{13} \right] + h.c.$$

where $L_\mu = i F^2 U^\dagger D_\mu U$

K decays (~50 channels) $\Rightarrow$ richest source of info about the mesonic sector of ChPT.

$$|G_8| \simeq 9 \cdot 10^{-6}\ \mathrm{GeV}^{-2}$$

$$G_{27}/G_8 \simeq 1/18$$

Radiative non-leptonic K decays

- $A(K \to [\pi]\, \gamma^* \cdots \gamma^*) = 0$ at $O(p^2)$

$K \to \pi\gamma\gamma \qquad K \to \pi\mu\mu$

$(K \to \pi\gamma^* \to \pi\mu\mu)$

- $A(K \to \pi\pi\, \gamma^* \cdots \gamma^*)$

$= A(\pi\pi) \cdot A_{brems.}$ at $O(p^2)$

$K_L \to \pi\pi \quad \cancel{CP}$

$K^+ \to \pi^+\pi^0 \quad \Delta I = \frac{1}{2}$ rule

$K \to \pi\pi\gamma$ at $O(p^4)$

$$A\left[K(p) \to \pi_1(p_1)\,\pi_2(p_2)\,\gamma(q,\epsilon)\right]$$

$$= \epsilon^\mu\left[\underline{E}\,(p_1\cdot q\,p_{2\mu} - p_2\cdot q\,p_{1\mu}) + \underline{M}\,\epsilon_{\mu\nu\alpha\beta}\,p_1^\nu p_2^\alpha q^\beta\right]/M_K^3$$

$$E = E_{IB} + E_{DE}$$

$E_{IB} \sim 1/E_\gamma^2$, $\quad E_{DE} = \text{const.}$ (small)

$$M_L^{(4)} = \frac{e G_8 m_K^3}{2\pi^2 F}\left[\quad a_2 + 2a_4\right]$$ const.

$$M_+^{(4)} = \frac{e G_8 m_K^3}{2\pi^2 F}\left[-1 + \frac{3}{2}a_2 - 3a_3\right]$$ const.

$U(1)_A$ anomaly

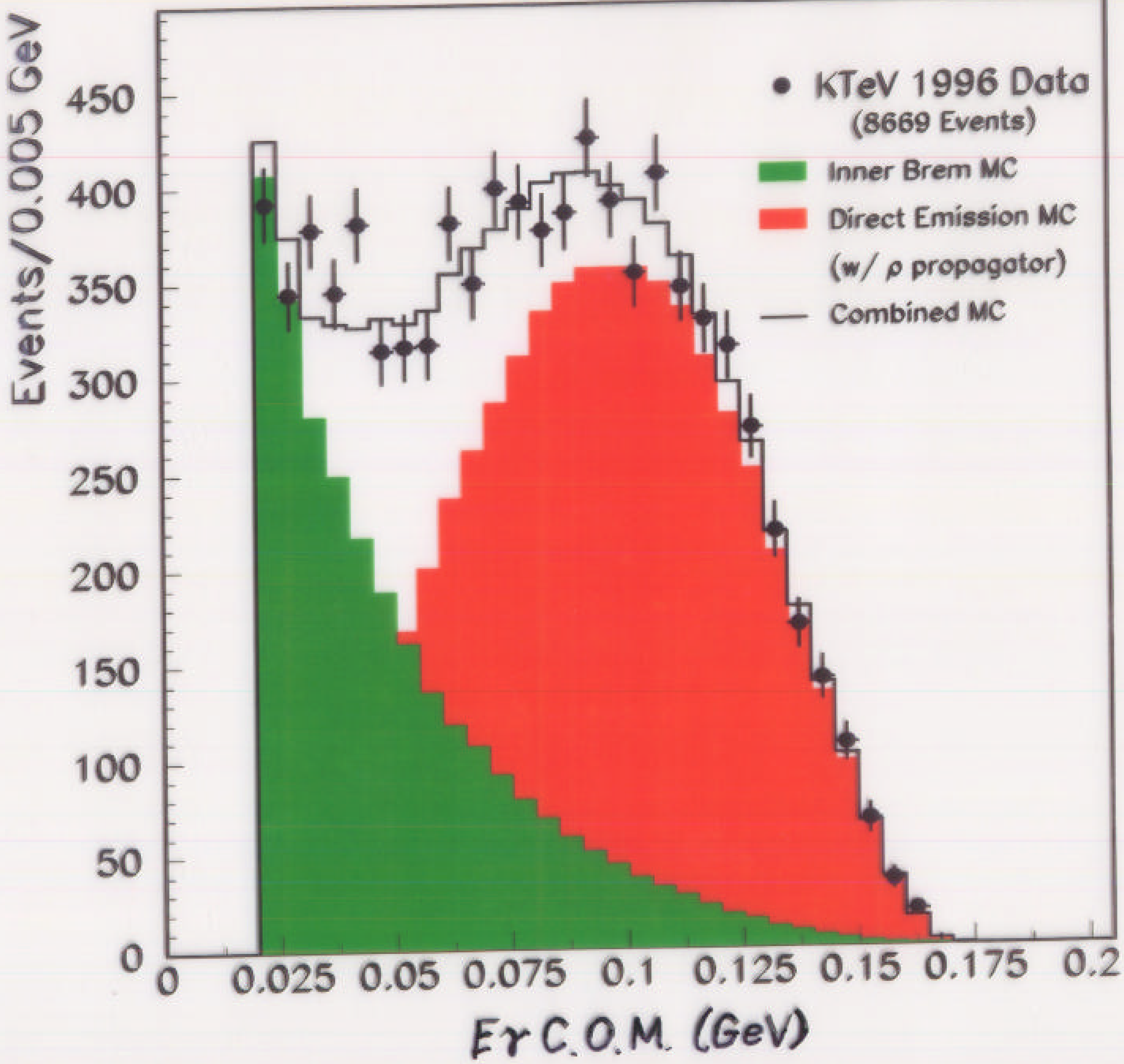
$$a_i \le 1$$

$$\frac{\tau_L}{B_{K_L\to\pi\pi\gamma}(DE)} = \frac{5.17\times 10^{-8}\,s}{3.10\times 10^{-5}} = 1.7\times 10^{-3}\,s$$

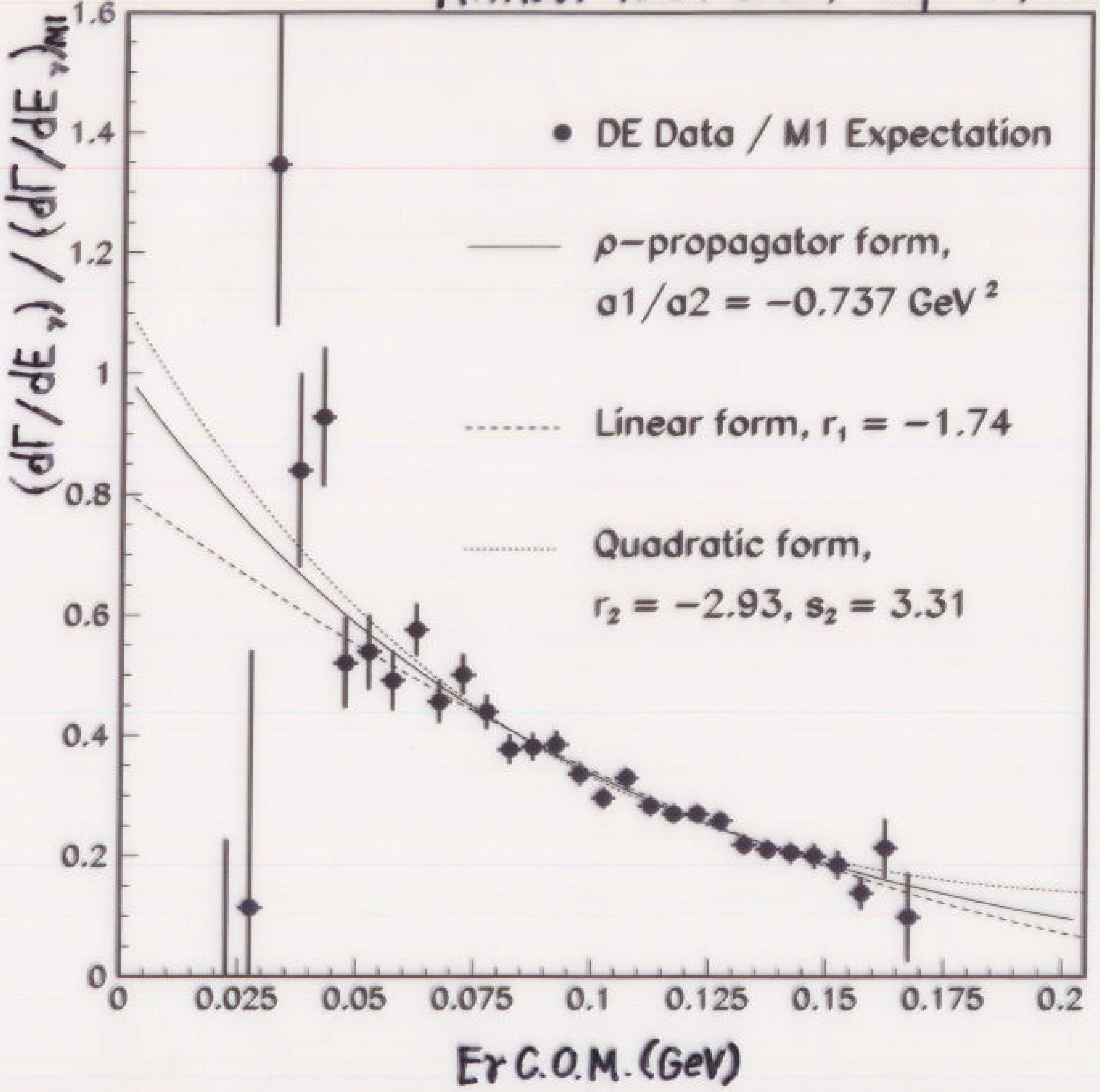
$$\frac{\tau_+}{B_{K_+\to\pi\pi\gamma}(DE)} = \frac{1.2\times 10^{-8}}{4.7\times 10^{-6}} = 2.6\times 10^{-3}\,s$$

A. Alavi-Harati et al hep-ex/0008045

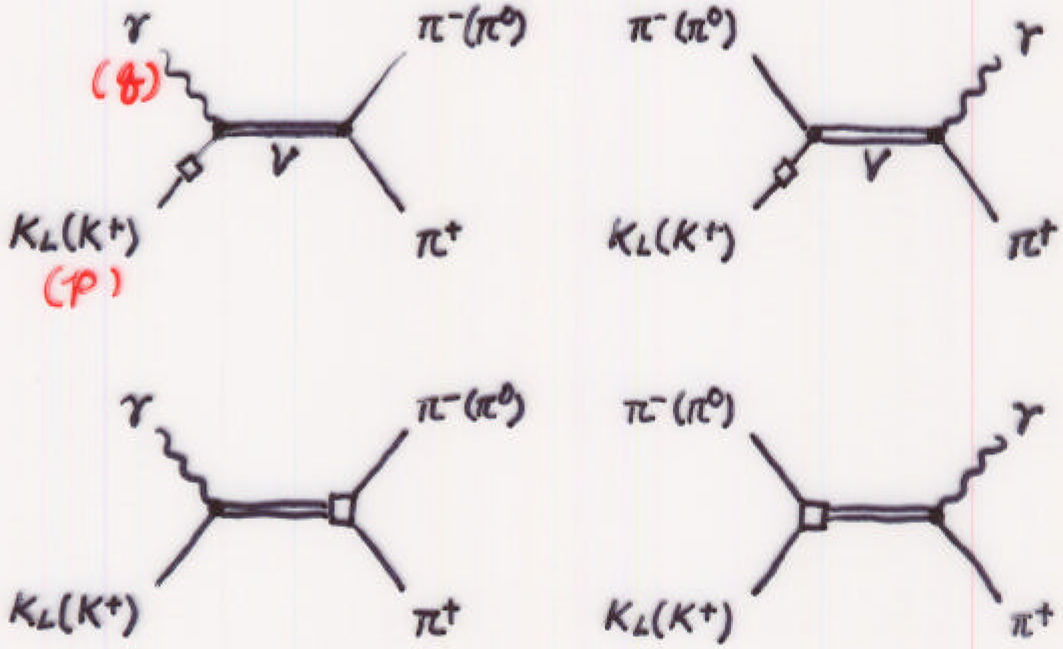
PRL 86 (2001) 761



A.Alavi-Harati et al, hep-ex/0008045



Vector Meson Dominance



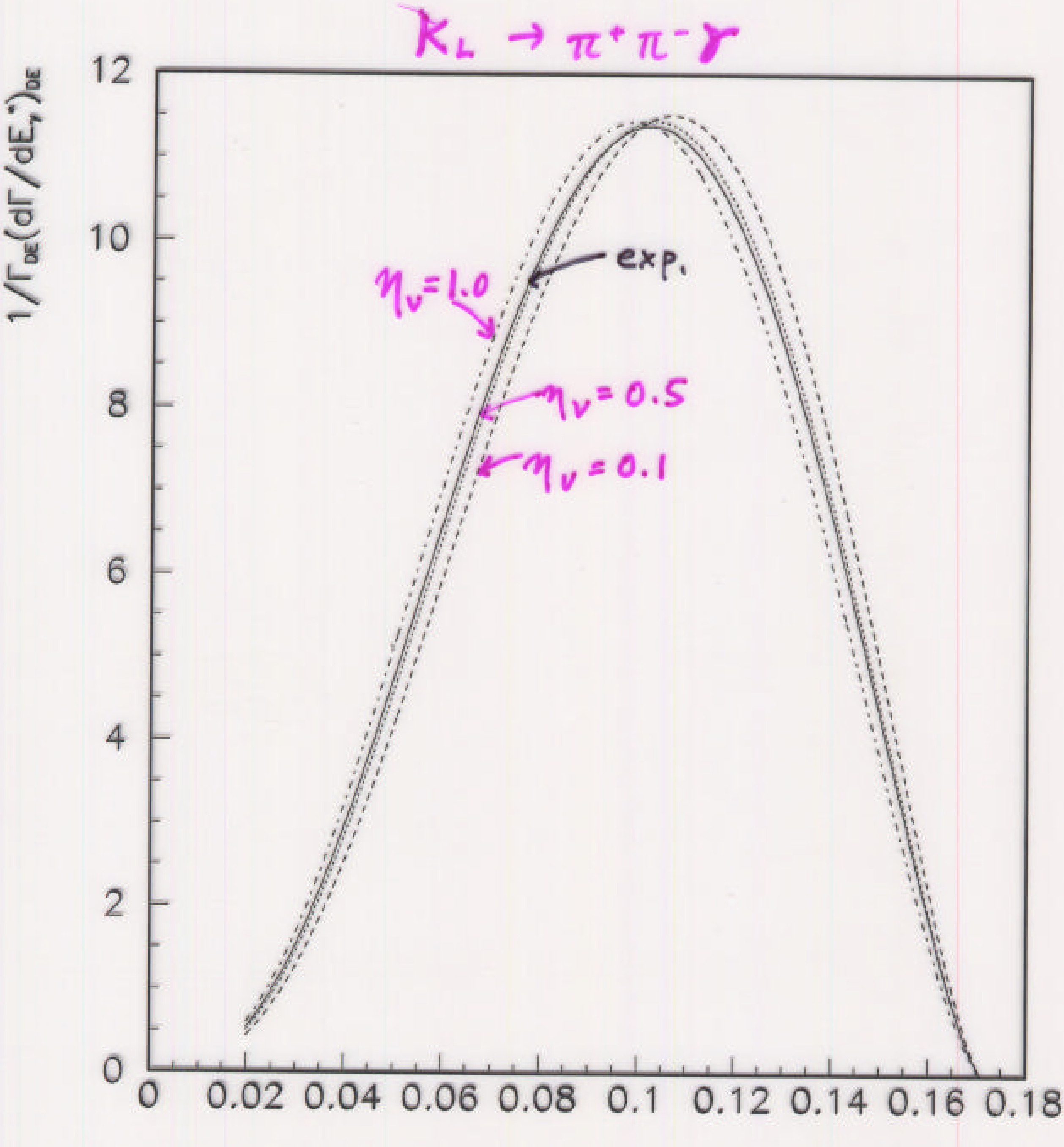
D'Ambrosio, Gao
hep-ph/0010122

$$M^L_{VMD} = \frac{eG_8 m_K^3}{2\pi^2 F}\,\tilde{q}\left[\frac{\eta_V + \frac{m_K^2}{m_V^2}(1-2Z_3)}{1-\frac{m_K^2}{m_V^2}+\frac{2m_K^2}{m_V^2}Z_3} + \frac{\frac{\eta_V}{2} - \frac{m_K^2}{m_V^2}Z_3}{1-\frac{m_K^2}{m_V^2}Z_3}\right]$$

$Z_3 = \frac{p\cdot q}{m_K^2}$ energy dependent

$\eta_V \simeq 0.3$ factorization model

(Nucl. phys. B492('97)417)



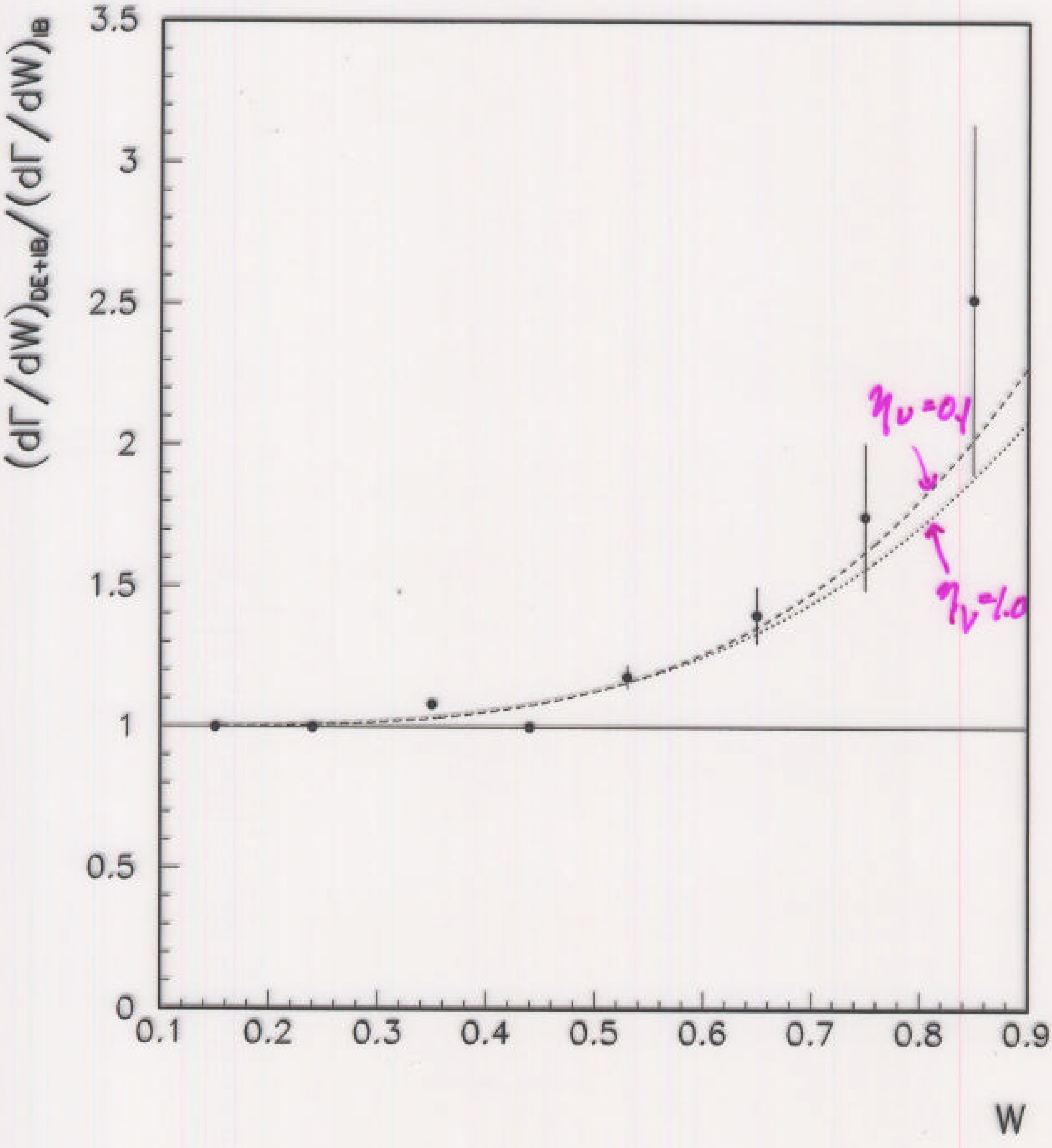
$K_L \to \pi^+\pi^-\gamma$
$1/\Gamma_{DE}(d\Gamma/dE_\gamma^*)_{DE}$
exp.
$\eta_v=1.0$
$\eta_v=0.5$
$\eta_v=0.1$
E_γ^*(GeV)

$$K^+(p) \longrightarrow \pi^+(p_+)\,\pi^0(p_0)\,\gamma(q)$$

$$\frac{\partial \Gamma}{\partial W} = \frac{\partial \Gamma_{IB}}{\partial W}\left[1 + \underbrace{2\frac{m_{\pi^+}^2}{m_K} \mathrm{Re}\left(\frac{E}{eA}\right)}_{\mathrm{INT}} W^2 + \underbrace{\frac{m_{\pi^+}^2}{m_K^2}\left(\left|\frac{E}{eA}\right|^2 + \left|\frac{M}{eA}\right|^2\right)}_{\mathrm{DE}} W^4\right]$$

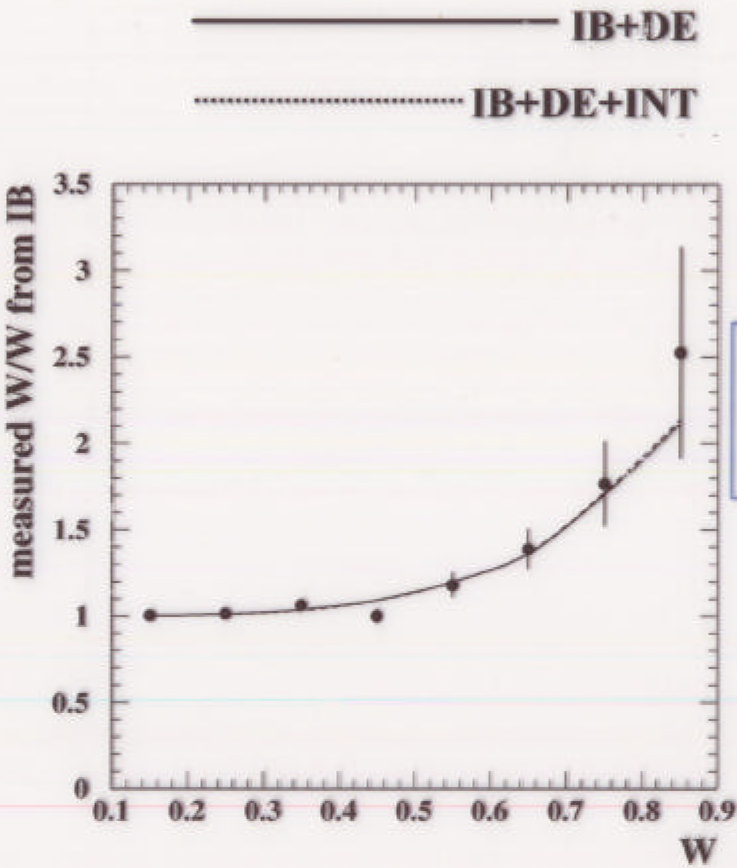
where

$$W^2 \equiv \frac{(p \cdot q)(p_+ \cdot q)}{m_{\pi^+}^2 m_K^2}$$



$(d\Gamma/dW)_{DE+IB}/(d\Gamma/dW)_{IB}$
3.5
3
2.5
2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
W
$\eta_V = 0.1$
$\eta_V = 1.0$

W Fitting with IB+DE+INT



χ^2/n.d.f = 7.9/5

χ^2/n.d.f = 7.8/5

R(INT/IB) = (-0.4 ± 1.6) × 10-2

-2.6 × 10-8 < Re(E) < 1.6 × 10-8

@67% C.L.

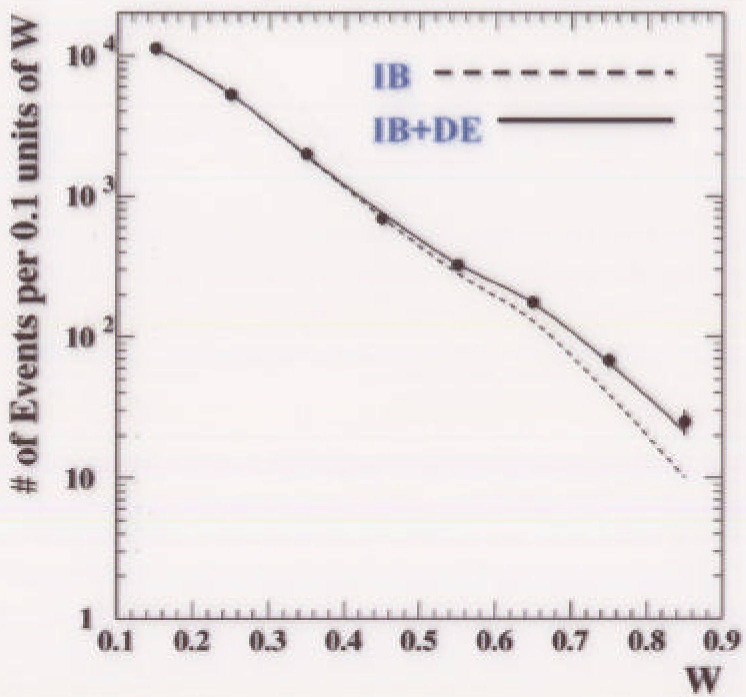
$|M| = (2.1 \pm 0.2) \times 10^{-7}$

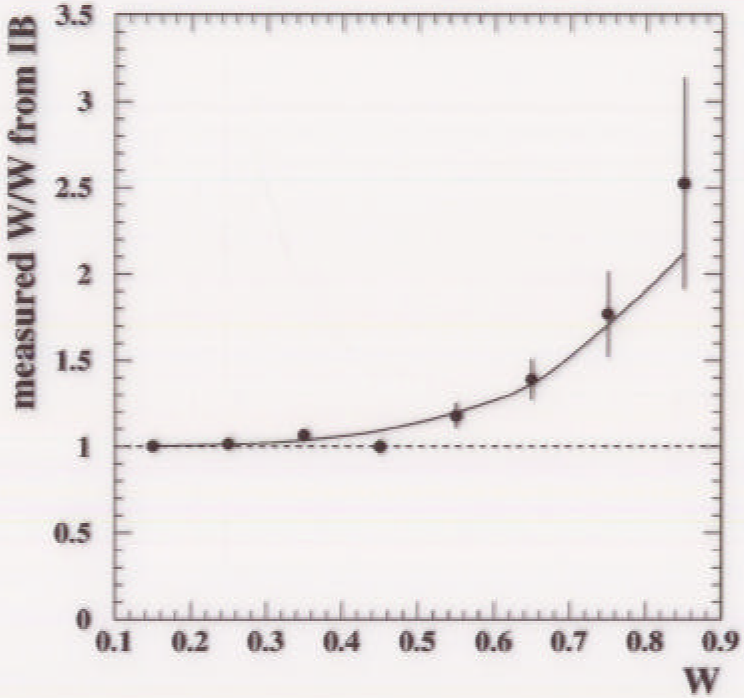
$\updownarrow$

$|M_{UZA}| = 1.8 \times 10^{-7}$

W Fitting Result

PRL 85('00) 4856
hep-ex/0007021





55MeV<T⁺<90MeV

$B(DE)/B(IB)=(1.8\pm0.3)\times10^{-2}$

$K^+(p)\to\pi^+(p_+)\,\pi^0(p_0)\,\gamma(q)$

$B(IB)=2.61\times10^{-4}$

$W^2=(qp)(qp_+)/(m^2_\pi m^2_K)$

$\boxed{B(DE)=(4.7\pm0.8)\times10^{-6}}$

Prospects

$K^+ \to \pi^+\pi^0\gamma$ with high stat. in a wide T^+ region

(KEK, BNL)

$\Downarrow$

VMD   INT(IB+E)

$K_L \to \pi^+\pi^-\gamma$
$K_L \to \pi^+\pi^- e^+e^-$

$K^+ \to \pi^+\gamma\gamma$
at low $M_{\gamma\gamma}$ region

(B.G. for $K^+ \to \pi^+\gamma$)

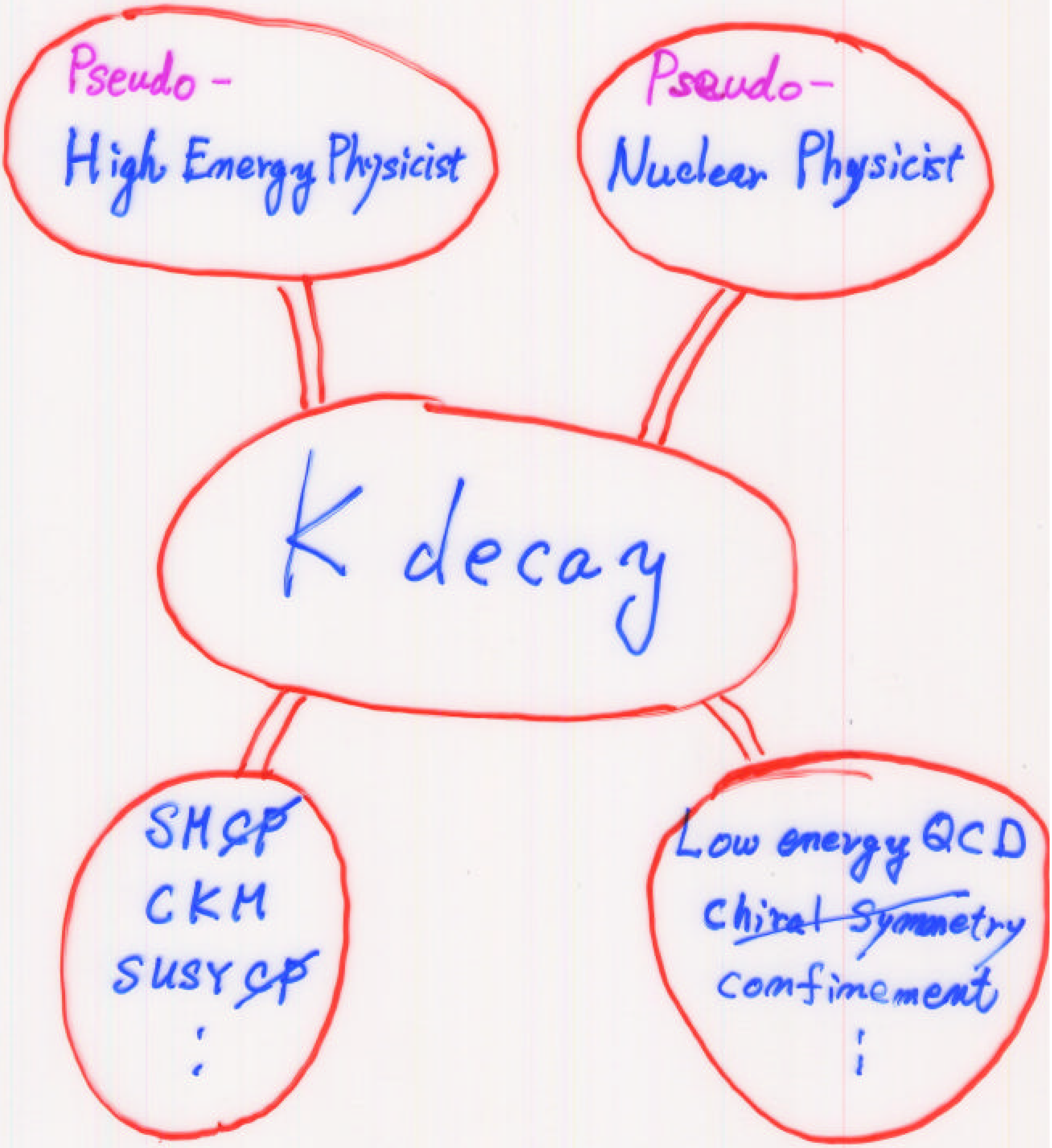
$K_S \to \pi^+\pi^-\gamma$

(CERN?)

SUSY $\not{CP}$

$\Gamma(K^+ \to \pi^+\pi^0\gamma) \neq \Gamma(K^- \to \pi^-\pi^0\gamma)$

(DAΦNE?)


Pseudo-
High Energy Physicist
Pseudo-
Nuclear Physicist
K decay
SM CP
CKM
SUSY CP
Low energy QCD
Chiral Symmetry
confinement