Letter of Intent: An Improved Muon $(g-2)$ Experiment at J-PARC

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Summary: The muon $(g-2)$ value has played an important role in constraining physics beyond the standard model. We propose to develop an improved Muon $(g-2)$ experiment at J-PARC with the design goal of an improvement of a factor of 5 to 10 over E821 at Brookhaven, (relative uncertainty 0.1 to 0.06 parts per million). At this proposed level of precision a robust potential exists to either limit new physics parameters or to determine their values. We propose to undertake the R&D necessary to reach this goal, and to move to J-PARC the pieces of the current experiment which would be useful to reaching this goal.

This Proposal Needs Fast Extracted Beam, 90 bunch extraction using the pulsed-proton beam extraction.

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Contents

1 Introduction

The muon $(g-2)$ experiment, E821 at the Brookhaven National Laboratory Alternating Gradient Synchrotron will reach a relative precision of about 0.6-0.5 ppm if no further data are collected. Results from stored μ^+ with 13, 5, 1.3 and 0.7 ppm precision have been published.[1, 2, 3, 4] One further data set on μ^- is being analyzed. While the improved precision now available represents a substantial improvement over the 7.3 ppm measurement from CERN,[5] it is clear that further improvement could be realized at J-PARC.

In this Letter of Intent (LOI) we present the improvements which we anticipate could be achieved in the next generation experiment at J-PARC. The goal is an improvement of a factor of 5 to 10 beyond the precision presently reached by E821, assuming that the forthcoming μ^- result is in agreement with the published μ^+ ones.

While substantial research and development will be necessary to achieve this new goal, we believe that it is possible, and such an experiment will continue to be of interest for its ability to restrict, or point to, physics beyond the standard model.

Below we first summarize the progress made to date at BNL. We then discuss the standard model contribution to $(g-2)$, and the issues which will be resolved over the next few years. We then identify the key issues which must be addressed for a new experiment to reach the goals we have set.

In this discussion we assume that the current $(g-2)$ storage ring and beamline will form a central part of any improved experiment. The BNL management has expressed its willingness to permit any $(g-2)$ related hardware to move to J-PARC.[6] This proposal would need to have the concurrence of the U.S. Department of Energy.

This LOI grows out of presentations at two different meetings in Japan, HIMUS99 at KEK and NP02 in Kyoto,[7, 8] and from discussions within the E821 collaboration as a whole.

2 Motivation and General Considerations

The measurement of magnetic moments has been important in advancing our knowledge of sub-atomic physics since the famous 1921 paper of Stern,[9] which laid out the principles of what we now call the "Stern-Gerlach experiment". The experimental and theoretical developments in the study of the electron's anomalous magnetic moment represent one of the great success stories of modern physics. The experiment has reached a relative precision of ~ 4 parts in 10^9 (parts per billion)[10] and the theory is constrained by our knowledge of the fine-structure constant α, rather than by the eighth-order and tenth-order QED calculations.[11]

The gyromagnetic ratio g is defined by

$$\vec{\mu}_s = g\left(\frac{e}{2m}\right)\vec{s}, \tag{1}$$

where $\vec{s}$ is the spin angular momentum, and $\vec{\mu}$ is the magnetic moment resulting from this angular momentum. The Dirac equation for point particles predicts that $g \equiv 2$. The muon magnetic moment consists of a Dirac part and an anomalous (Pauli) part a. Thus for a positive muon,

$$\mu = (1+a)\frac{e\hbar}{2m} \qquad \text{where} \qquad a\frac{e\hbar}{2m} = \frac{(g-2)}{2}\frac{e\hbar}{2m} \tag{2}$$

is the anomalous magnetic moment (or simply the anomaly).

At the present precision of the electron $(g-2)$ measurements, the result can be described by QED with photons and electrons only. For the muon, the relative contribution of heavier particles enters as

$$\left(\frac{m_\mu}{m_e}\right)^2 \simeq 40,000 \tag{3}$$

so the muon anomaly is sensitive to the effects of heavier virtual particles at a measurable level.

Since this LOI is a companion to the LOI to use the PRISM facility to search for the lepton flavor violating process $\mu \to e$, and the LOI to measure the muon electric dipole moment (edm), we will connect these three separate lines of investigation here, before focusing on the muon anomalous magnetic dipole moment. We use the example of supersymmetry (*SUSY*) to show the connection between these three processes, although *SUSY* need not be the "true" extension to the standard model. While $(g-2)$ has a relatively large standard model value which is dominated by the one-loop QED contribution, the standard-model expectation for a muon electric dipole moment, which would violate both P and T symmetries separately,[1] is many orders of magnitude smaller than that expected from *SUSY*, or other extensions to the standard model.

The muon magnetic anomaly a_μ and the edm are related to each other as the real and imaginary parts of related operators,[12, 13, 14] which can be seen from their definitions:

$$\mathcal{L}_{mdm} = a_\mu \frac{e}{4m_\mu}\overline{\mu}\sigma^{\alpha\beta}F_{\alpha\beta} \tag{4}$$

$$\mathcal{L}_{edm} = -\frac{i}{2}d_\mu\overline{\mu}\sigma^{\alpha\beta}\gamma_5\mu F_{\alpha\beta} \tag{5}$$

where the indices α, β run from 0 to 3, and d_μ is the electric dipole moment.

The relevant *SUSY* diagrams are shown in Fig. 1. The $(g-2)$ value and the edm probe a diagonal matrix element of the slepton mixing matrix, while muon conversion probes an off-diagonal one. If the present $(g-2)$ experiment were to

[1](along with *CP* if *CPT* is conserved)

observe a significant deviation from the standard model, it would be interpreted as favoring a range of $SUSY$ masses and values of $\tan\beta$. In the event that the $SUSY$ mass spectrum is subsequently measured at LHC, then muon $(g-2)$ will provide one of the cleanest measurements of $\tan\beta$, while the edm measurement will provide the new CP-violating phase

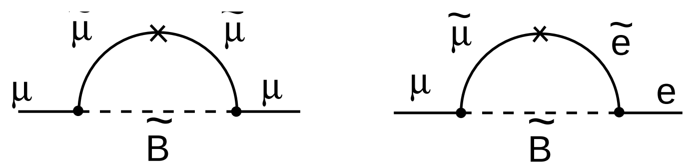

Figure 1: The supersymmetric contribution to $(g-2)$ and $\mu \to e$.

It is clear that all three of these processes must be measured as accurately as possible to provide information on physics beyond the standard model. The standard model contribution to $(g-2)$ must be known accurately to interpret any $(g-2)$ result in the context of physics beyond the standard model. The standard model value will be discussed in detail below, but we note at this point that a better understanding of the hadronic contribution is the subject of intense effort at a number of facilities.

3 Summary of the Brookhaven Experiment

We propose to use the method used in the BNL experiment, where polarized muons were injected into the storage ring with a fast kicker. This experiment is based on the fact that for $a_\mu > 0$ the spin precesses faster than the momentum vector when a muon travels transversely to a magnetic field. The Larmor and Thomas spin-precession and the momentum precession frequencies are

$$\omega_S = \frac{geB}{2mc} + (1-\gamma)\frac{eB}{\gamma mc}; \qquad \omega_C = \frac{eB}{mc\gamma} \tag{6}$$

and the difference frequency gives the frequency with which the spin precesses relative to the momentum,

$$\omega_a = \omega_S - \omega_C = (\frac{g-2}{2})\frac{eB}{mc} = a_\mu \frac{eB}{mc} \tag{7}$$

which is proportional to the anomaly, rather than to the full magnetic moment. A precision measurement of a_μ requires precision measurements of the precession frequency ω_a and the magnetic field, which is expressed as the free-proton precession frequency ω_p in the storage ring magnetic field.

In E821 we use an electric quadrupole field to provide vertical focusing, and the precession frequency becomes

$$\vec{\omega}_a = -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta} \times \vec{E}\right]. \tag{8}$$

and for the "magic $\gamma = 29.3$" the electric field does not change the spin direction relative to the momentum. We use this "magic γ in E821, and propose to use it also at J-PARC.

A muon bunch is injected into the storage ring. The subsequent high-energy decay positrons (electrons) are detected, and their energies and arrival times are measured. The time spectrum of high-energy positrons shows the muon exponential decay modulated by the $(g-2)$ spin precession in the storage ring.

Data Set	*Statistical Error* (ppm)	*Systematic Error* (ppm)	*Status*
1999 μ^+	1.25	0.5	Published, 10^9 events
2000 μ^+	0.6	0.43	Published, 4×10^9 events
2001 μ^-	~ 0.7	~ 0.3	Projected, $\sim 3 - 4 \times 10^9$ events

Table 1: The $(g-2)$ data sets obtained at Brookhaven. For comparison, the CERN experiment reported on about 10^8 events with an average decay asymmetry A^2 about half that obtained in E821. The 2001 error is a projection from our ongoing analysis.

In Table 1 we summarize the statistical and systematic errors thus far. Since the systematic and statistical errors are independent, the total error is the quadrature of the two.

In Table 2 the final statistical errors are summarized, both for the published data,[1, 2, 3, 4] and a projected one for the μ^- data set collected in 2001.

Data Set	*# of Events*	*Statistical Error (ppm)*
Total μ^+	5×10^9	0.56
Total present μ^-	$\sim 3 \times 10^9$	~ 0.7
Total present μ^+ & μ^-	8×10^9	~ 0.44

Table 2: The combined statistical errors on the data sets.

4 Status of the Theory of Muon $(g-2)$

The theoretical value of $a_\mu = (g_\mu - 2)/2$ in the standard model has its dominant contribution from quantum electrodynamics but the weak and strong interactions contribute as well.

$$a_\mu(\mathrm{SM}) = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{weak}) + a_\mu(\mathrm{had}) \tag{9}$$

The current values are[16]

$$a_\mu(\mathrm{QED}) = 11\ 658\ 470.57(0.29) \times 10^{-10}(0.025\ \mathrm{ppm}) \tag{10}$$
$$a_\mu(\mathrm{weak}) = 15.1(0.4) \times 10^{-10}(0.03\ \mathrm{ppm}) \tag{11}$$

While the QED and weak contributions are quite well known in comparison with the error goals of this LOI, the hadronic contribution is not. The evaluation of the lowest-order hadronic contribution is currently the object of both theoretical and experimental investigation, as is the hadronic light-by-light (lbl) contribution, both of which are discussed in detail below.

The term a_μ(QED) is obtained using the value of α from a_e(exp)=a_e(SM), and terms through order α^5 are included.[11, 15, 16] The small uncertainty arises from the coefficient of the α^4 term is being re-evaluated.[17]

The term a_μ(weak) arises from virtual radiative processes dominantly involving the Z and W bosons, while the Higgs boson does not contribute significantly for $m_H > 100$ GeV.[26, 27, 28, 29, 30] The value a_μ(weak) given in Eq. 11 includes electroweak contributions of up to two-loop order. The 3-loop electroweak leading log terms have been estimated to contribute 0.5×10^{-11} which is considerably smaller than the uncertainty in a_μ(weak).

4.1 The Hadronic Contributions

The leading order contribution from hadronic vacuum polarization $a_\mu(\mathrm{Had};1)$, is determined primarily from the dispersion relation

$$a_\mu(\mathrm{Had};1) = (\frac{\alpha m_\mu}{3\pi})^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) R(s) \tag{12}$$

in which

$$R \equiv \frac{\sigma_{\mathrm{tot}}(e^+e^- \to \mathrm{hadrons})}{\sigma_{\mathrm{tot}}(e^+e^- \to \mu^+\mu^-)}, \tag{13}$$

is obtained from experiment, and K is a kinematic factor.

When the present experiment was beginning at Brookhaven in the very early 1980s, the hadronic contribution was known to about 6.4 ppm.[5, 18, 19] By the mid-1980s, in anticipation of an experiment which could observe the electroweak contribution, the uncertainty was improved to 1.5 ppm.[20] With the interest generated by our experiment, this value is now known to about 0.6 ppm,[21, 22, 23, 24] as discussed below.

This dispersion integral is dominated by the low-energy data from Novosibirsk, which has recently published much higher precision data,[31] which cover the range from threshold to $\sqrt{s} = 1.4$ GeV. Also important are new data from the BES collaboration in Beijing. These data, which cover from $\sqrt{s} = 2$ GeV to 5 GeV,[32] are in agreement with the QCD calculations of Davier and Hockër[21] in the region down to 1.8 GeV. The earlier data with much larger uncertainties tended to lie above the QCD prediction.

The quantity a_μ(had 1) can also be evaluated from τ decay data as indicated in Fig. 2(b), assuming CVC. Such data from ALEPH have been used by DH98.[21]

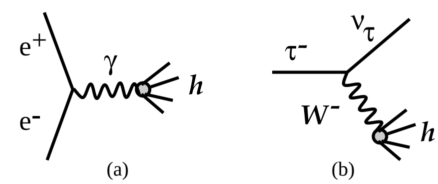

Figure 2: (a). The hadroproduction process which enters the dispersion relation. (b) Hadronic τ decay.

The leading order contribution $a_\mu(\mathrm{Had};1)$ contributes the largest uncertainty to a_μ(SM). Until recently, $a_\mu(\mathrm{had},1) = 692(6) \times 10^{-10}$ (0.6 ppm) was the most reliable value,[21] where data from both hadronic τ-decay and e^+e^- annihilation were used to obtain a single value for $a_\mu(\mathrm{had},1)$. Recently, two new evaluations[23, 24] using the new e^+e^- results from Novosibirsk[31] have become available, and Ref. [23] also uses data from hadronic τ-decay. The two new analyses of e^+e^- data agree with each other quite well. However, with the much larger τ-decay data sample now available, the value of $a_\mu(\mathrm{had},1)$ obtained from τ-decay does not agree with the value obtained from e^+e^- data over the relevant range of $\sqrt{s}$.[23]

The higher-order hadronic contributions include[33] $a_\mu(\mathrm{had},2) = -10.0(0.6) \times 10^{-10}$, and the now agreed upon positive sign for the contribution from hadronic light-by-light scattering gives[34] $a_\mu(\mathrm{had,lbl}) = +8.6(3.2) \times 10^{-10}$. Using the published value of $a_\mu(\mathrm{had},1)$ from Ref. [21], the standard model value is $a_\mu(\mathrm{SM}) = 11/659\,177(7) \times 10^{-10}$ (0.6 ppm). From the most recent measurements at BNL,[4] the muon anomalous magnetic moment is $a_\mu(\mathrm{exp}) = 11\,659\,203(8) \times 10^{-10}$ (± 0.7 ppm). The difference between a_μ(exp) and a_μ(SM) above is about 2.6 times the combined statistical and theoretical uncertainty. If the new e^+e^- evaluations are used[23, 24] the discrepancy is ~ 3 standard deviations, and using the τ-analysis alone over the appropriate energy region[23] gives a 1.6 standard deviation discrepancy. These results are shown graphically in Fig. 3.

The significance for an indication of new physics will have to wait for clarification of the correctness of the hadronic contribution. The theoretical path from e^+e^-

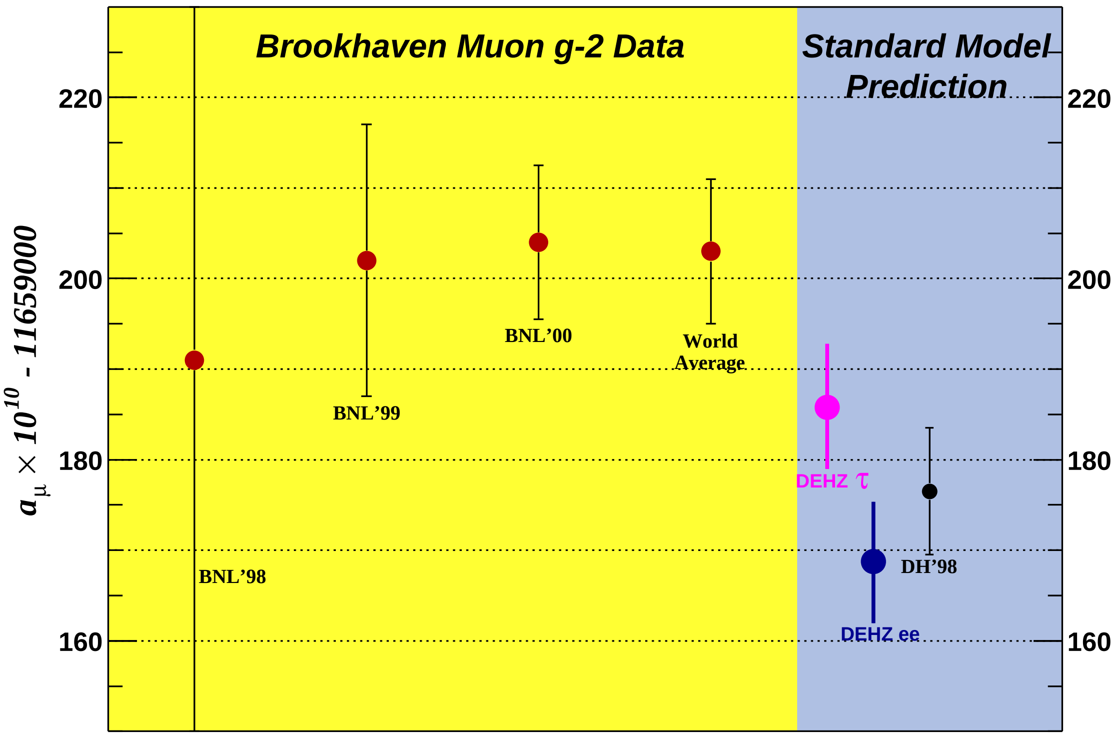

Figure 3: E821 measurements of a_μ carried out with direct muon injection into the storage ring. The relative uncertainties are ± 5 ppm (98), ± 1.3 ppm (99), ± 0.7 ppm (00), where the number in parentheses is the year when the data were collected. All measurements are for μ^+. The three theory points use the lowest order hadronic contribution from DH98[21] and the separate DEHZ evaluations[23] from τ and e^+e^- hadronic data. The DH98 evaluation includes both τ and e^+e^- data.

data to a value of $a_\mu(\text{had}, 1)$ is more straightforward than from the τ-decay data. Since the new data from Novosibirsk dominate the determination of the lowest order hadronic contribution, it is essential that these data be checked. At higher energy colliders, the cross section can be determined over a range of cm energies by using e^+e^- collisions where initial state radiation of a single photon lowers the center of mass energy of the collision. In the literature this process is referred to as "radiative return", or alternately initial state radiation (ISR). Substantial activity exploiting this technique is currently under way at Frascatti,[35] SLAC[36] and KEK[37] to check the Novosibirsk e^+e^--data, and the technique shows considerable promise.

In summary, a substantial amount of work, both theoretical and experimental, is on-going at a number of institutes across the world to further refine the value of the hadronic contribution to $(g-2)$. We have every confidence that if this experiment goes ahead, the theory will continue to improve toward sufficient accuracy.

5 Future Improvements at Brookhaven?

To reach the ultimate sensitivity, it is clear that J-PARC is unique both in flux, and in the large harmonic ($n = 90$) available for operation. E821 is officially completed.[38] Given the large investment already in place at Brookhaven, we are exploring what improvement might be realized there, if interest were to develop in an improved experiment. Such a program would be complementary to what could be accomplished at J-PARC, since the ultimate precision would only be possible at J-PARC. Whatever improvements in hardware developed for an improved experiment at the AGS, would certainly be used in the final configuration at J-PARC.

6 The Experiment at J-PARC

A number of serious issues would need to be confronted to realize the projected improvement at J-PARC. If no further work is possible at Brookhaven, the current experimental apparatus, with small improvements could form the core of the first effort at J-PARC, with a program of improvements and upgrades which would lead to the ultimate precision possible there.

To reach the goal of 0.06 ppm, major improvements to all systems would be necessary. A new beamline, which could use many of the components of the E821 beamline would needed, with a lithium lens or magnetic horn plus additional quadrupoles to increase the capture efficiency. A substantial effort must be made to increase the inflector aperture, especially in the radial direction.

The precision magnetic field would have to be further refined, especially concerning calibration issues. The detectors and electronics would have to be upgraded to permit higher counting rates with less sensitivity to pileup. Both new detectors and waveform digitizers would be necessary. Since a proton and a muon precession frequency are needed to determine a_μ, we use the current systematic errors as a basis for discussion of the improvements needed to achieve a goal of 0.06 ppm.

6.1 The Beamline, Inflector and Kicker

The E821 beamline consists of a production target which can receive up to 7×10^{12} protons (tera protons or Tp) in a $\sigma = 25$ ns pulse from the AGS. Pions produced at zero degrees are collected by a quadrupole doublet, and then momentum selected. Since the beam hitting the target is an order of magnitude larger (per hour) than we had at Brookhaven, the whole production target system will need serious study.

Muons are produced in a 72 m long fodo decay channel. At the end of the decay channel, a second momentum slit is set to 1-2% lower than p_π, which permits the selection of forward muons with a polarization of about 95%. In E821 this produces a beam which is about 50% muons. The $\pi^\pm$ $e^\pm$ and proton (antiproton) contamination produces background which is detected by the calorimeters, causing baseline shifts at injection, and pulse pileup. While an improved experiment could begin data collection with the same beam conditions as are present in E821, one would need to have a well defined plan to upgrade over the next several years to obtain a pure muon beam. An obvious possibility for improving beam purity is to use a backward muon beam. This would require doubling the length of the decay channel, which might not be feasible. Another possibility is multiple momentum slits with large-angle bends. We will explore all of these to optimize the purity at a minimum increase in complexity of the beamline.

If the emittance of the inflector could be improved,[39] the beamline could be redesigned to improve the injection efficiency, which is about 7% in E821. Calculations and simulations show that the phase-space mismatch drops the efficiency to 20% (see Fig. 4), and multiple scattering off the closed ends of the inflector magnet drops it to about 9%. The less than ideal kicker pulse drops the injection efficiency further to 7.3%. Thus an inflector with one or both ends open, and with a larger beam aperture could in principle lead to large improvements in storage efficiency.

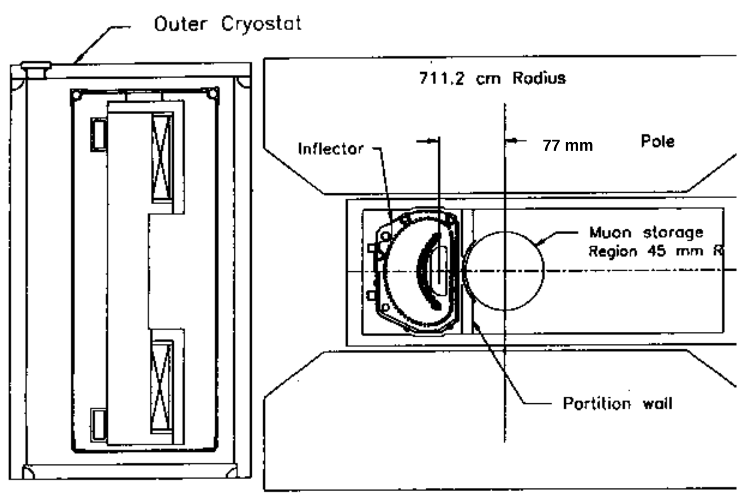

Figure 4: The inflector exit-vacuum chamber geometry. The center of the storage ring is to the right. The gap between the pole pieces is 180 mm, and the inflector exit is 18 × 56 mm^2 (ignoring the chamfer on the outer radius corners).

The kicker pulse is formed by a simple LCR circuit. A capacitor is charged to 95 kV, and then a deuterium thyratron shorts the capacitor to ground. The principal issue with the kicker waveform is that it is too wide, since the minimum inductance which was achieved was 1.8 μH. There may be improvements which could be made to decrease the inductance, but a new idea would be needed to reduce it by much.

The existing $(g-2)$ fast kicker[40] could be pulsed for 90 bunch extraction. The main improvement needed is to provide forced circulation of the oil through the region of the resistor stack with a cooling device to remove the heat.

6.2 Muon Rates

In E821, a proton bunch with $\leq$ 7 Tp strikes a Ni target. Because of the shock delivered by the beam on the target, it is unlikely that this bunch intensity can be exceeded with a simple solid target. With the projected J-PARC intensity, and 90 bunches, the bunch intensity is approximately 7 Tp. This increase in bunch number gives approximately a factor of ten in the data rate over the BNL experiment.

To improve on the E821 result by a factor of 10, implies a factor of 100 in data, or on the order of $\sim 8 \times 10^{11}$ muons which produce detected electrons or positrons above 2 GeV. This implies a few tera μ stored. The total running time at the AGS, neglecting end effects, and set up, was about 7 to 8 months calendar time. Since 70 months running time at J-PARC is not possible, it will be necessary to improve other factors to get the additional data. A lithium lens, or pulsed magnetic horn should improve things by around a factor of 5. An improved beam line, and better phase space matching through the inflector might gain an additional factor of 3 to 4. The ultimate statistical uncertainty will depend on our ability to improve on beam related factors, and to re-design the detectors and electronics to handle this increase in rates. Pileup and beam backgrounds must be kept to a minimum, especially by improving the muon beam purity substantially over what was possible with the E821 beamline.

6.3 Systematic Errors on ω_a

The systematic errors from the 1999 and 2000 data sets are given below in Table 3.

Pulse pileup occurs when two electrons overlap in time a single calorimeter, and cannot be recognized by the pulse-finding algorithm as two separate pulses. There are three possibilities: (a). Two pulses, each of which would have been below the 2 GeV software threshold, overlap and are accepted; (b) Two pulses which would have been above the threshold can pile up, producing a single higher energy pulse; (c) One pulse below hardware threshold can pile up with another one such that the combined pulse is over the software threshold. All of these cases will have the wrong energy and $(g-2)$ phase. Furthermore, there is "unseen pileup" which is pileup of a pulse of "reasonable" size with a small pulse which does not appear in our measured pileup spectrum. Since there are a large number of these small pulses, and we cannot measure them, our largest pileup uncertainty comes from them. Triple pileup is also

Source of Systematic Error	*1999 Error* (ppm)	*2000 Error* (ppm)
Pileup	0.13	0.13
AGS Background	0.10	-
Lost Muons	0.10	0.10
Timing Shifts	0.10	0.02
E field and vertical betatron motion	0.08	0.08
Binning and fitting	0.07	0.06
Coherent betatron Oscillations	0.20	0.21
Beam Debunching/randomization	0.04	0.04
Gain Changes	0.02	0.13
Total systematic Error on $< \omega_a >$	0.3	0.31
Statistical Error on ω_a	1.3	0.62

Table 3: The systematic errors on ω_a from the 1999 and 2000 data sets.

observed, but is at a much smaller level than double pileup, and thus is not included as a separate effect in the analysis.

It was possible to construct a pileup spectrum, by looking at some fixed time $\Delta t = 10$ ns $> \delta t$ after the first pulse found in a waveform digitizer record, where δt is the pulse resolving time. If a pulse was found in the window $\Delta t \pm \delta t$ then it was added to the first pulse to obtain the pileup spectrum. A second method for constructing the pileup spectrum was to vary the resolving time in software, and to construct a pileup spectrum.

In the analysis, pileup was dealt with in two different ways: subtraction before fitting, or included as parameters in the fit with the pileup phase determined from the generated pileup spectrum. Both methods gave similar results, and the value obtained for ω_a from the different analyses did not depend on which pileup treatment was used. Nevertheless, pileup continues to be an issue in the data analysis. The new experiment must have detectors which are less sensitive to pileup, presumably with substantial segmentation. At present, we have 4 tubes on the calorimeter, but only two real segments, top and bottom.[41] We would need a 3×3 or perhaps 4×4 array to help reduce the pileup sensitivity. Faster waveform digitization will also be needed, and the issue of "unseen pileup" must also be addressed. Since the data volume will be substantially larger than we have on the present experiment, we cannot just lower the threshold. However, a system to sample this low-energy pileup will be needed. A calorimeter with greater segmentation, increased light output, and faster waveform digitization should improve the systematic error due to pileup substantially.

As early as the 1997 pion injection run, we observed that at times protons were extracted from the AGS during the 700 μs$(g-2)$ measurement time. This is a potential problem, since the AGS cyclotron frequency characteristic of this background could mix with the $(g-2)$ precession frequency. While this background could be minimized with proper timing of the AGS extraction orbit-bump and kicker magnet, the level of this background was still unsatisfactory. In the 2000 run, a pulsed sweeper magnet was added to the secondary beamline which reduced this background to negligible levels. We also changed our monitoring of these AGS after pulses by turning off the quadrupoles every 25th fill of the ring to look directly for these pulses. This sweeper magnet would also be needed at J-PARC, but presumably an upgrade of the present system should suffice.

We store beam for about 4000 turns (700 μs) in the ring. At injection into the ring, the beam is displaced off center and scraped for 16 μs, and then returned to the center of the storage region. Nevertheless, real equilibrium is never reached, and muons are lost during the entire storage time. Vertically segmented hodoscopes are in front of some of the calorimeters. We monitor lost muons by looking for coincidences between hodoscopes in three adjacent detector stations, with no visible energy in the corresponding calorimeters. This produces a rather clean spectrum of lost muons, which we use to determine the time constant (lifetime) phase and level of these losses. All versions of the multi-parameter fits include a muon loss term. Better understanding of the muon losses will be necessary at J-PARC.

Timing shifts within a fill of the ring, i.e. from early to late in the fill, could introduce a serious systematic error. We now find that this systematic error is 0.02 ppm with the present detectors. This specification must be held to in the next generation detectors.

Effects from the coherent motion of the beam, which we call CBO (coherent betatron oscillations) were discovered in our 1999 data set. Since the acceptance of the detectors is a function of the radial position of the muon when it decays, the decay electron/positron spectrum is modulated by the CBO frequency. With the improved statistics now available, we see additional issues associated with the CBO. To study these effects, we varied the field index (high voltage on the electrostatic quadrupoles) in the 2001 run, which enabled us to study the effects of the CBO frequency on the $(g-2)$ frequency. Changing the n-value also changed the muon loss rate, permitting us to learn more about the effect of muon losses from the storage ring. Several suggestions have been made on how to decrease our sensitivity to the CBO, or how to eliminate the CBO, but these ideas need further study.

Additional suggestions and details for improving on both ω_a and ω_p are given in the write-up by Miller from NP02.[8]

6.4 Systematic Errors on ω_p

The storage ring[42] magnetic field is measured using the proton spin precession frequency in the same field region where the muons are stored, which is proportional to the field. Pulsed NMR[43] is applied in which the free induction decay of protons in water is determined. There are 375 NMR probes outside of the vacuum chamber next to the magnet pole pieces. Of these, about 150 are used to monitor the field during data collection. Several times a week, the beam is turned off, and an NMR trolley travels through the vacuum chamber mapping the field. The fixed probes are calibrated by the map obtained from the trolley. Before and after the running period, the trolley probes are calibrated with a probe[44] which has a spherical water sample, and gives a calibration relative to the free-induction frequency of the unbound proton.[45]

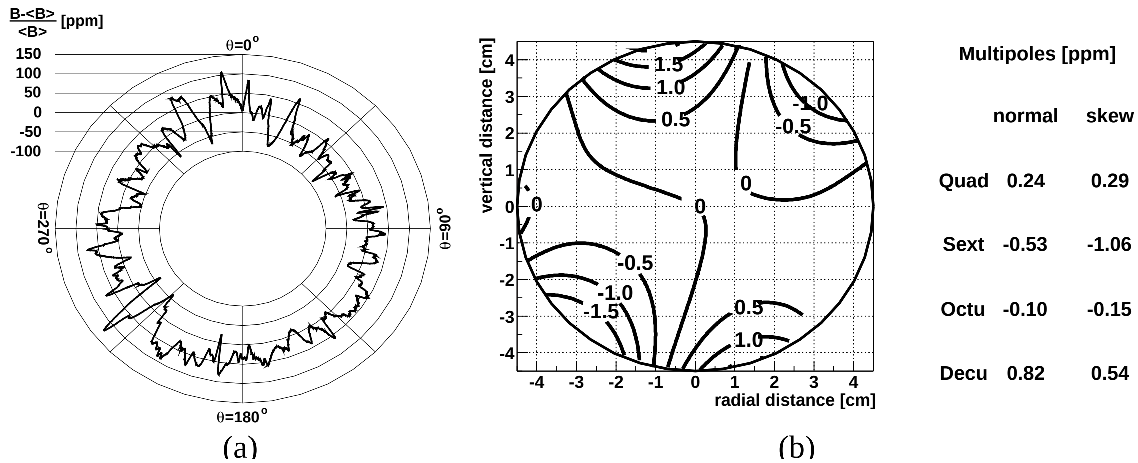

Figure 5: (a) The deviation from the average field value in ppm measured at the center of the storage region, as a function of azimuth for one of the 22 trolley measurements. (b) A 2-dimensional multipole expansion of the field averaged over azimuth from one out of 22 trolley measurements. Half ppm contours with respect to a central azimuthal average field $B_0 = 1.451\,274\,\mathrm{T}$ are shown. The multipole amplitudes relative to B_0, are given at the beam aperture, which has a radius of 4.5 cm and is indicated by the circle.

In Fig. 5 one field map is shown, along with a multipole decomposition from another trolley run. The field map shows the difference in ppm from the average field, as a function of azimuth.

The dominant errors from the field measurement are given in Table 4. The problem from the inflector fringe field was eliminated by replacing the first inflector, which was damaged during initial testing, between the 1999 and 2000 running period, removing this source of error. This improvement had the additional benefit that the average field is more uniform, thus lowering our sensitivity (and systematic error) to our knowledge of the muon distribution.

Source of Systematic Error	*1999 Error* (ppm)	*2000 Error* (ppm)
Inflector Fringe Field	0.20	-
Calibration of trolley probes	0.20	0.15
Interpolation with fixed probes	0.15	0.10
Trolley measurements of B_0	0.10	0.10
Uncertainty from μ-distribution	0.12	0.03
Absolute Calibration of standard probe	0.05	0.05
Others†	0.15	0.10
Total systematic Error on $<\omega_p>$	0.4	0.24

Table 4: The systematic errors on ω_p from the 1999 and 2000 data sets. The absolute calibration of the spherical probe refers to the calibration of the spherical water probe to the free proton. Calibration of the trolley probes refers to the two-step process of: (i) calibrating the trolley probes relative to a plunging probe which is moved into a position to measure the same field measured by each of the trolley probes, along with (ii) the calibration of the plunging probe relative to the spherical water probe.
†Higher multipoles, trolley temperature and its power supply voltage response, eddy currents from the kicker.

The deviation from the average field versus azimuth, along with a typical multipole decomposition of the field in the storage region is shown in Fig. 5

For a new experiment, all aspects of the field would need to be improved. Better shimming at the pole-piece boundaries, would be needed, and one would probably re-machine the pole pieces to a flatter tolerance, before starting. Once the magnet was re-assembled at J-PARC, an entire new program of shimming would need to be carried out. Much of our experience in shimming the present magnet would be useful, and would help expedite the new shimming.

If a new inflector magnet is installed, it would need to have flux leakage controlled at least as well as in the present one, and perhaps better.

Eddy currents from the kicker would have to be monitored carefully using a Faraday effect magnetometer,[40] and if they are not at a negligible level, one would have to correct for them offline.

The NMR absolute calibration depends on the measurements of Phillips et al.,[45] Winkler et al.,[46] plus calculations,[47] which give the calibration of the NMR frequency in a spherical water sample relative to the free proton. While the accuracy of this measurement is fine for the present $(g-2)$ experiment, an alternative or complementary calibration technique will need to be found. We are considering the use of a ^3He magnetometer to complement the spherical water sample, and while expertise exists to develop this technique, it will require development work.[48]

The magnetic field which enters in Eq. 8 is the field averaged over the muon distribution. Thus we will have to have information on the B-field distribution and the muon distribution to extract a_μ from the data. This will be improved substantially by a more uniform magnetic field, which is more stable in time. In E821 the temperature of the ring is not well stabilized. At J-PARC, we would need good temperature stability for the ring, (1 degree C) and we would need improved shimming of the magnet. As can be seen from Fig. 5 the azimuthal field variations are seen to be $\pm \sim 60$ ppm from the central value. If this could be reduced substantially, the contribution to the systematic error from the muon distribution will be negligible.

6.5 Measurement of μ_μ/μ_p

To obtain a_μ from the experiment, the fundamental constant $\lambda = \mu_\mu/\mu_p$ must be known to an adequate precision. The present best direct measurement of λ (± 120 ppb) is from a muonium ground-state hyperfine-structure experiment at LAMPF.[49] A "more precise" value for this ratio can be obtained using theoretical calculations, which has an uncertainty of (± 25 ppb). Since we believe that to rely on theory for this important number might be risky, we feel that an improved version of the LAMPF experiment should be done. It could be started at at PSI, where we will have available the pulsed beam being developed by the MuLAN collaboration and the microwave system which was used at LAMPF (which is now at KVI). While substantial progress could be made at PSI, to reach the ultimate precision it would be necessary to move the experiment to a more intense pulsed muon source.[50] The 10 ns wide proton bunch, combined with the pulsed-proton beam extraction needed by the PRISM facility at J-PARC would be the obvious choice. If the J-PARC muon $(g-2)$ experiment goes ahead, a subset of the collaboration would pursue this measurement just as the Yale and Heidelberg groups pursued the previous measurement at LAMPF[49] in the years before E821 collected data.

7 Resources: From and To J-PARC

In this section, we list what we will need from J-PARC and what we might expect to bring. This list should not be viewed as complete, but it contains the major items. As mentioned early on, the Associate Director for High Energy and Nuclear Physics at Brookhaven has expressed his willingness to permit us to bring $(g-2)$ related equipment to J-PARC, however the U.S. Department of Energy (DOE) would have to agree to this plan. Furthermore, we would expect to request funding to support our participation from the U.S. National Science Foundation, and the U.S. DOE including funds for new hardware. Having put these caveats forward first, we now detail what we would need, and expect to bring.

While we are exploring alternate ways of improving on E821, we assume that the present storage ring would be useful for the J-PARC experiment, and we propose to move it along with its power supply to J-PARC. The 14 m diameter superconducting coils will provide a challenge to move, but we assume that if the DOE agrees, that a way will be found to move them. The refrigerator which we used needs to be replaced. We had an adequate refrigerator obtained from SSC Lab allocated to the experiment, but it was never installed. Whether the Laboratory would permit that system to leave is uncertain, and would depend on potential future use by the laboratory for approved BNL projects. The superconducting inflector, vacuum chambers, and other mechanical components would certainly be available, but vacuum pumps, and other general purpose components would most likely be absorbed back into the Laboratory's pool. We would propose to bring the beamline elements, as well as a production target station, if it were desirable. Beamline power supplies might not be available, and might have to be provided by J-PARC. The pulsed beam sweeper magnet would be available along with its pulsed power supply, both of which would need to be upgraded.

We will request the funds needed to develop and build the new electronics and detectors which are needed for the experiment, along with the precision magnetic field hardware. Substantial R&D will be necessary for the field measurement and control, and we expect to do that work and support it within the collaboration.

From J-PARC we need the pulsed-proton beam extraction system with the option of 90 bunches, extracted one at at time with at least 1 ms spacing between extractions. We will need help in upgrading the beamline, including a lithium lens or magnetic horn, and power supplies for the beamline. The laboratory will need to take responsibility for the production target. Even if a target station is available with the beamline, a spare will need to be fabricated, in the event of failure. As mentioned above, the target station at J-PARC will need significant study.

In addition to the beamline, which is on the order of 100 m, we need a hall large enough to contain the 14 m diameter ring with services, and a substantial overhead crane. The ring will need temperature stabilization as discussed above. A counting room close to the experimental area would also be needed. A possible layout of the pulsed-proton beam extraction area is given in Fig. 6

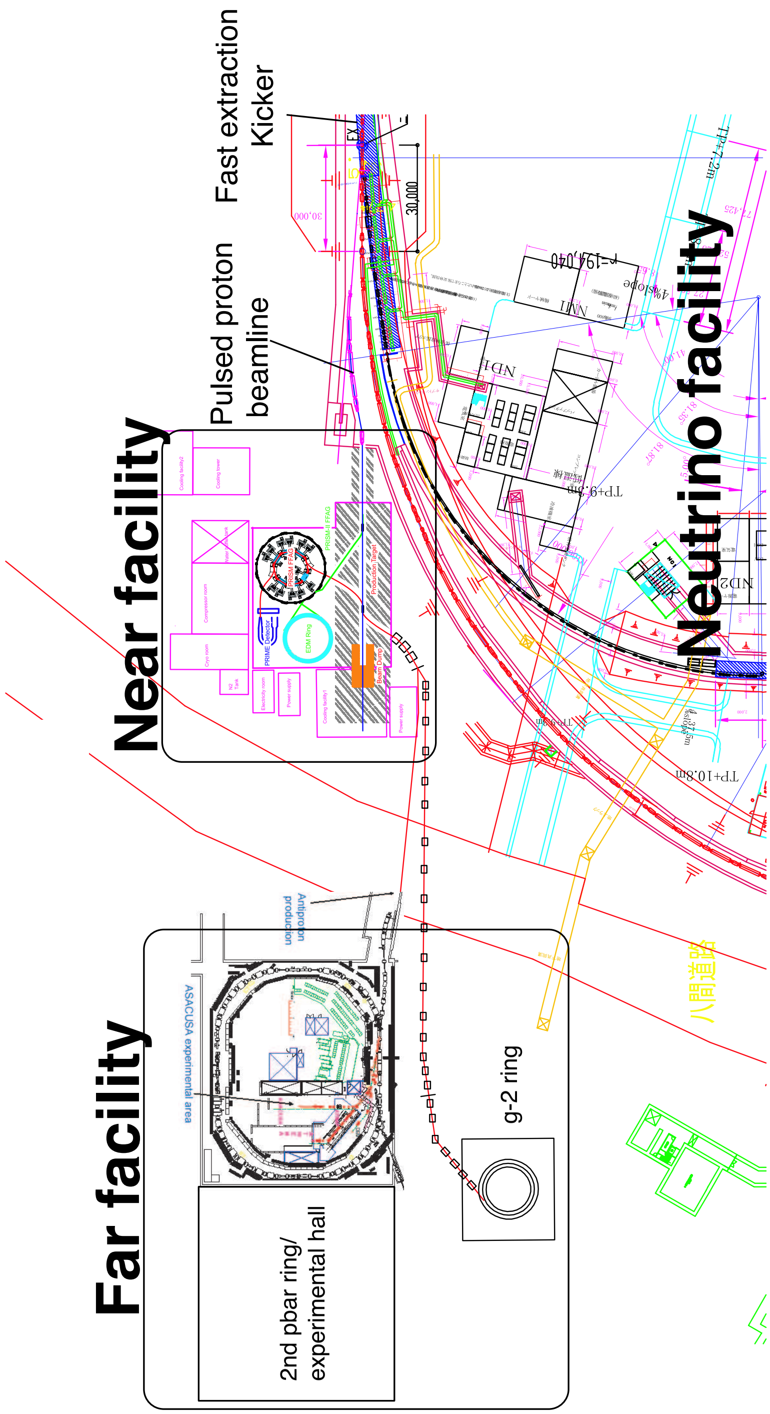

Figure 6: The layout of the pulsed-proton beam extraction and the $(g-2)$ storage ring.

8 Summary and Conclusions

We believe that an improved muon $(g-2)$ experiment can be mounted at J-PARC, which could reach an accuracy of ~ 0.10 to 0.06 ppm relative error. It appears that the present experimental apparatus (production target, beamline, inflector, kicker and storage ring) form the core of this experiment, but substantial research and development will be necessary to achieve these goals. At present, it appears to us that with considerable work the necessary specifications for the precision magnetic field and the measurement of the muon precession frequency can be achieved. We have learned much from our experience on E821, and just as E821 benefitted from the CERN experiment, the J-PARC measurement will be guided by our experience at Brookhaven.

The theory situation will certainly be clarified over the next few years. The model followed by E821, where we performed the best measurement possible and by our progress stimulated substantial interest and work on many aspects of the $(g-2)$ theory, seems reasonable for a future experiment. Already progress is being made on many fronts, and with adequate motivation theorists and experimentalists will continue to press forward on these problems.

The full significance for implications from the present $(g-2)$ data on new physics will have to wait for clarification of the correctness of the hadronic contribution. Nevertheless, a recent very conservative evaluation of the impact of $(g-2)$ on the constraining of supersymmetry parameters shows that even with the current uncertainties, $(g-2)$ already rules out a "substantial region of (susy) parameter space... that has not been probed by any previous experiment".[51]

Traditionally $(g-2)$ has served a valuable role in constraining new theories, and there is every indication that this role will continue into the future.

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