Four-, Five-, and Six-Body Calculations of Double Strangeness s-Shell Hypernuclei

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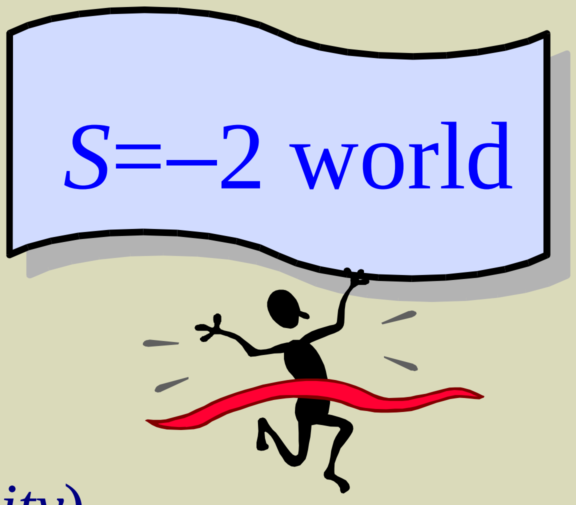
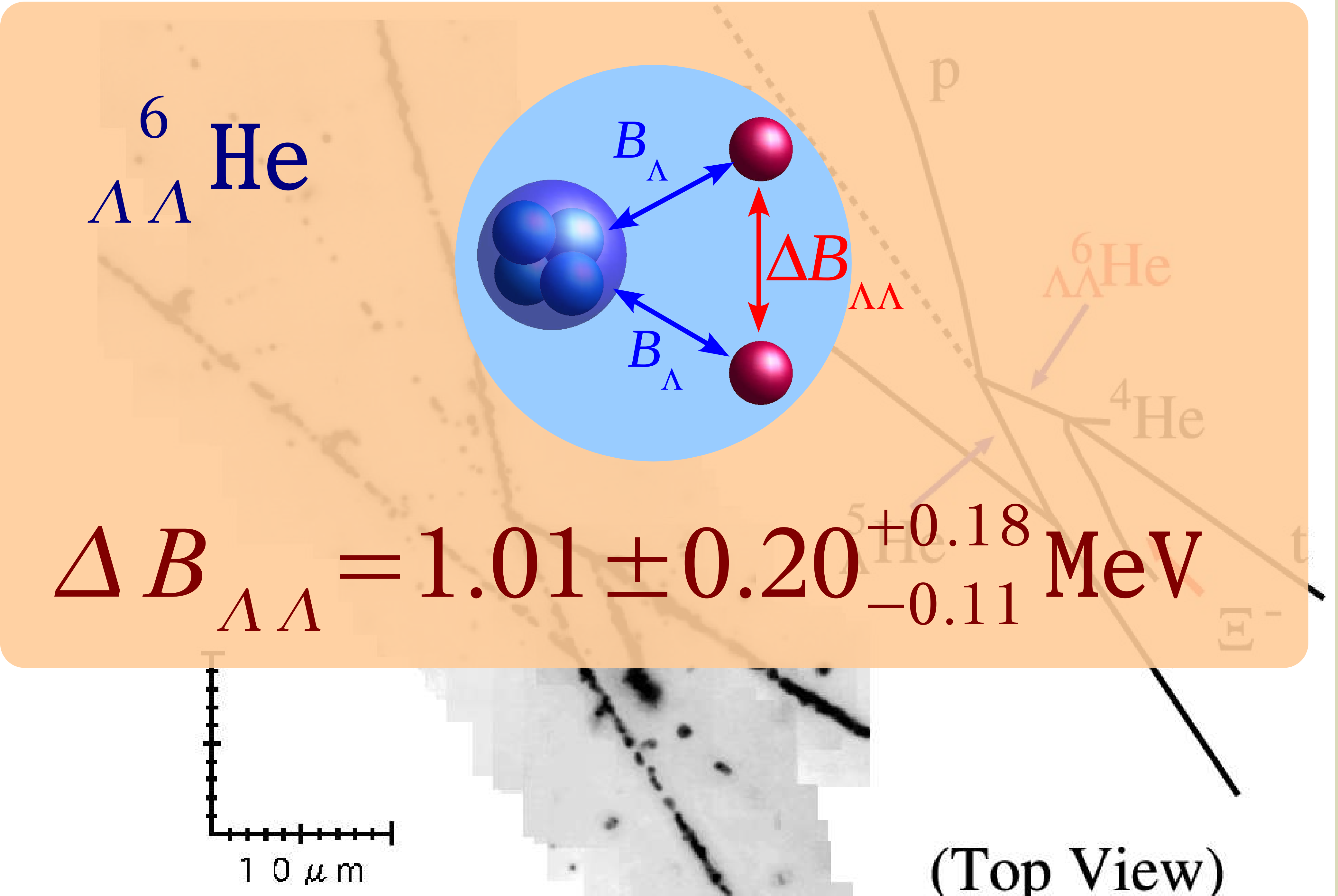


This talk is based on the work
nucl-th/0407033
in collaboration with
S. Shinmura (*Gifu University*)
Y. Akaishi (*Nihon University*)
Khin Swe Myint (*Mandalay University*)

"NAGARA" Event

$^{6}_{\Lambda\Lambda}He$



$$\Delta B_{\Lambda\Lambda} = 1.01 \pm 0.20^{+0.18}_{-0.11}\ \mathrm{MeV}$$

Introduction:

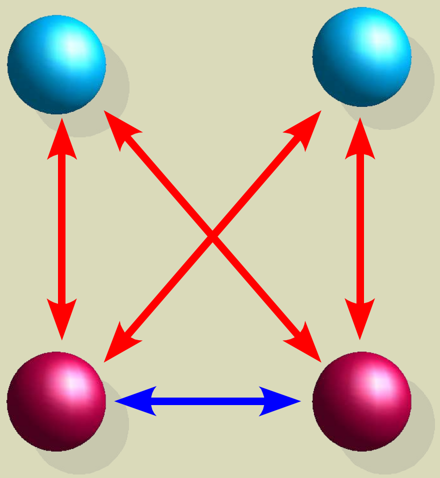
- ${}_{\Lambda\Lambda}{}^{6}$He: A door to the multistrangeness world
  - $\Delta B_{\Lambda\Lambda}$ ~ 4-5 MeV (Old data) [Prowse, PRL **17**, 782 (1966)]
  - $\Delta B_{\Lambda\Lambda}$ ~ 1 MeV (Nagara event) [Takahashi *et al*., PRL **87**, 212502 (2001)]
- ${}_{\Lambda\Lambda}{}^{4}$H: Is there a bound state?
  - Earlier theoretical predictions ➜ positive
    - Nakaichi-Maeda and Akaishi, PTP **84**, 1025 (1990).
    - H. N. *et al*., PTP **103**, 929 (2000).
  - BNL-AGS E906 experiment; formation of ${}_{\Lambda\Lambda}{}^{4}$H (?)
    - Ahn *et al*, PRL **87**, 132504 (2001).
  - A theoretical study of weak decay modes from ${}_{\Lambda\Lambda}{}^{4}$H ➜ negative
    - Kumagai-Fuse and Okabe, PRC **66**, 014003 (2002).
  - Faddeev-Yakubovsky search for ${}_{\Lambda\Lambda}{}^{4}$H (based on Nagara datum)
  - Filikhin and Gal, PRL **89**, 172502 (2002). ➜ negative? (but positive on $d\Lambda\Lambda$ model)

Introduction:

- Faddeev-Yakubovsky search for ${}_{\Lambda\Lambda}^{4}\mathrm{H}$
  - Filikhin and Gal, PRL **89**, 172502 (2002). ➜ negative? (but positive on $d\Lambda\Lambda$ model)
- Stochastic variational search for ${}_{\Lambda\Lambda}^{4}\mathrm{H}$
  - The result strongly depends on the choice of ΛN interaction.
- What is the problem on theoretical search for ${}_{\Lambda\Lambda}^{4}\mathrm{H}$?
  - Our publicatioin concluded that " *A theoretical search for ${}_{\Lambda\Lambda}^{4}\mathrm{H}$ is still an open subject*," because the " *${}^{3}S_{1}$ ΛN interaction has to be determined very carefully, since $B_{\Lambda\Lambda}$ is sensitive to the ${}^{3}S_{1}$ channel of the ΛN interaction*."
- How to determine the ${}^{3}S_{1}$ ΛN interaction?

Introduction: S=-2 hypernucleus

- A key issue of S=–2 study: The total binding energies of the S=–2 hypernuclei strongly depend on the strength of the ΛN interaction than the strength of the $\Lambda\Lambda$ interaction.
  - For example, ${}_{\Lambda\Lambda}{}^{4}\mathrm{H} \sim pn\Lambda\Lambda$
  - Number of ΛN pairs: 4
  - Number of $\Lambda\Lambda$ pairs: 1



Introduction:

- How to determine the $^3S_1$ ΛN interaction?
  - A detailed analysis concerning Λp scattering has not yet become available.
  - Experimental B_Λ values for ${}_{\Lambda}^{4}$H* , ${}_{\Lambda}^{4}$He* and ${}_{\Lambda}^{5}$He would give useful information for pinning down the $^3S_1$ ΛN interaction.
- However, there is a long standing problem on *s*-shell Λ hypernuclei: anomalously small binding of ${}_{\Lambda}^{5}$He.
  - Recently, Akaishi *et al*. successfully resolved the anomaly by explicitly taking account of ΛN-ΣN coupling.
    - Akaishi *et al*., PRL **84**, 3539 (2000).

Anomalously small binding of ${}^{5}_{\Lambda}$He

- Why is the ${}^{5}_{\Lambda}$He anomaly important for the study of S=–2 hypernuclei?
  - The algebraic structure of σ·σ part of ΛN potential for S=–2 hypernuclei is same as that for ${}^{5}_{\Lambda}$He.

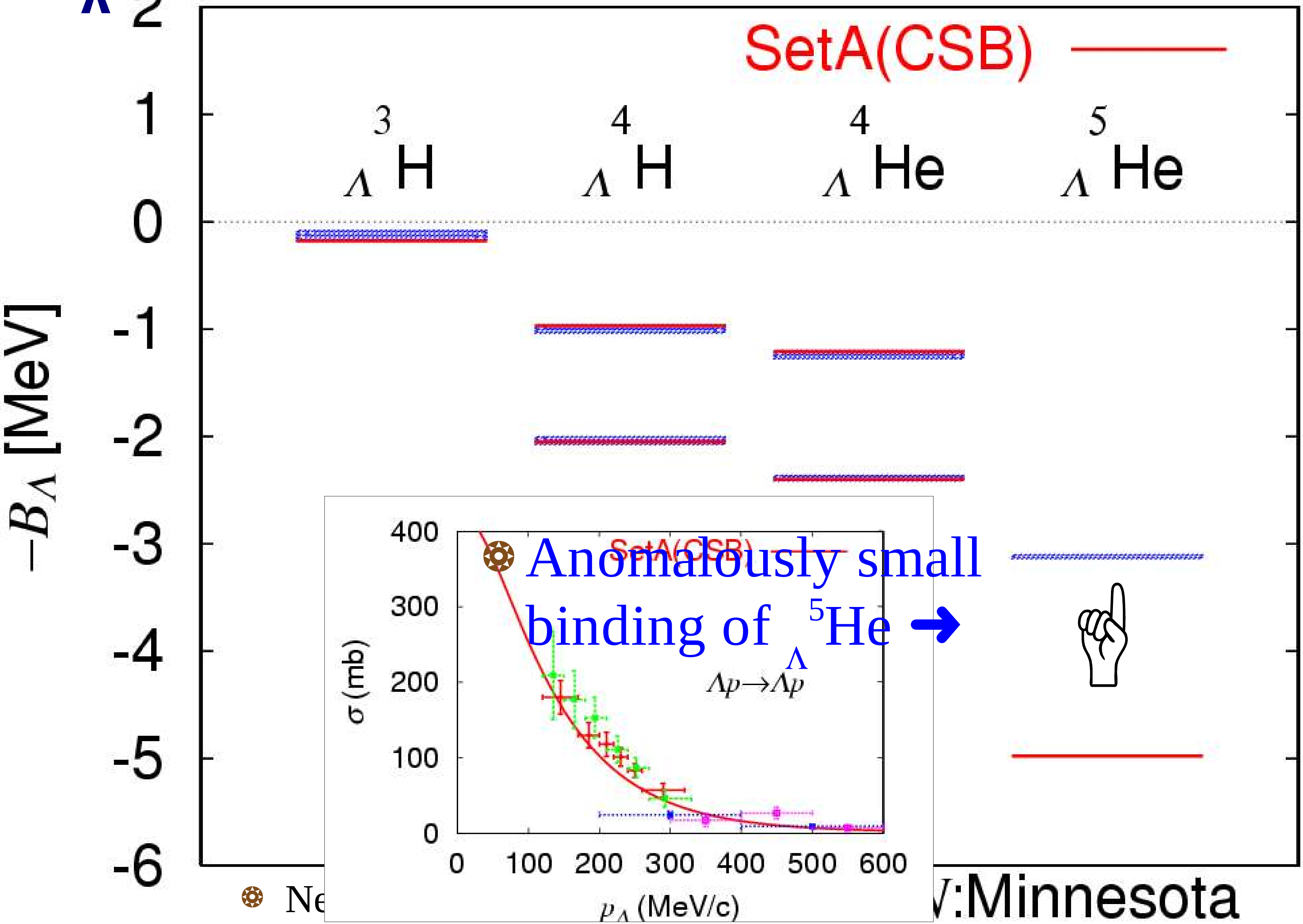
S=–1 hypernuclei

	$\left\langle \sum_{i=1}^{N} V(N_i \Lambda) \right\rangle$
${}^{3}_{\Lambda}$H	$\frac{1}{2}\bar{v}_t + \frac{3}{2}\bar{v}_s$
${}^{4}_{\Lambda}$H, ${}^{4}_{\Lambda}$He	$\frac{3}{2}\bar{v}_t + \frac{3}{2}\bar{v}_s$
${}^{4}_{\Lambda}$H*, ${}^{4}_{\Lambda}$He*	$\frac{5}{2}\bar{v}_t + \frac{1}{2}\bar{v}_s$
${}^{5}_{\Lambda}$He	$3\bar{v}_t + \bar{v}_s$ ⇇3:1

S=–2 hypernuclei

	$\left\langle \sum_{i=1}^{N} \sum_{j=1}^{Y} V(N_i \Lambda_j) \right\rangle$	$\langle V(\Lambda_1 \Lambda_2) \rangle$
${}^{4}_{\Lambda\Lambda}$H	$3\bar{v}_t + \bar{v}_s$	$\bar{u}_s$
${}^{5}_{\Lambda\Lambda}$H, ${}^{5}_{\Lambda\Lambda}$He	$\frac{9}{2}\bar{v}_t + \frac{3}{2}\bar{v}_s$	$\bar{u}_s$
${}^{6}_{\Lambda\Lambda}$He	$6\bar{v}_t + 2\bar{v}_s$	$\bar{u}_s$
	⇈3:1 ⇈	

Anomalously small binding of ${}^{5}_{\Lambda}He$

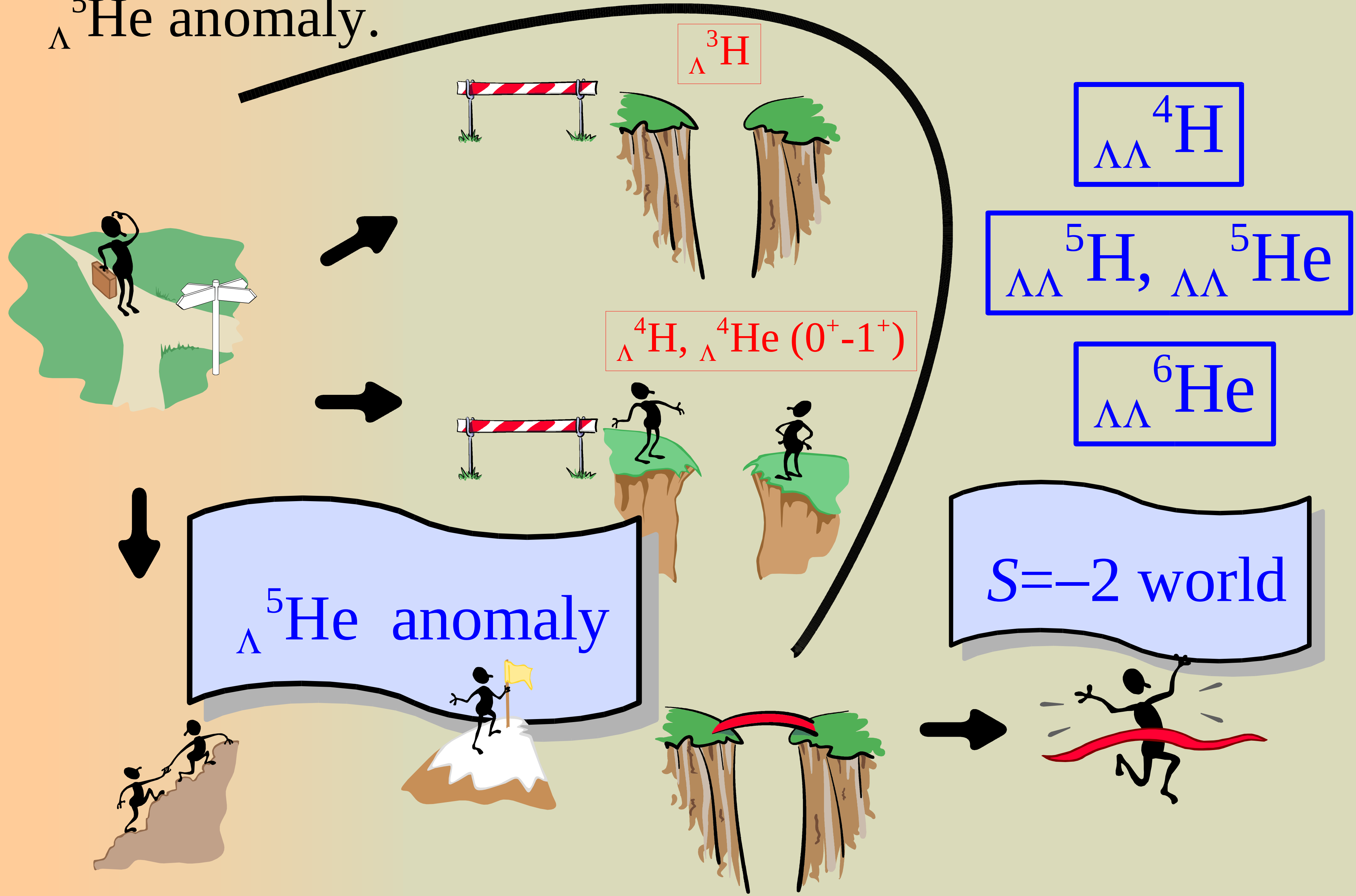


- Anomalously small binding of ${}^{5}_{\Lambda}He$ ➜

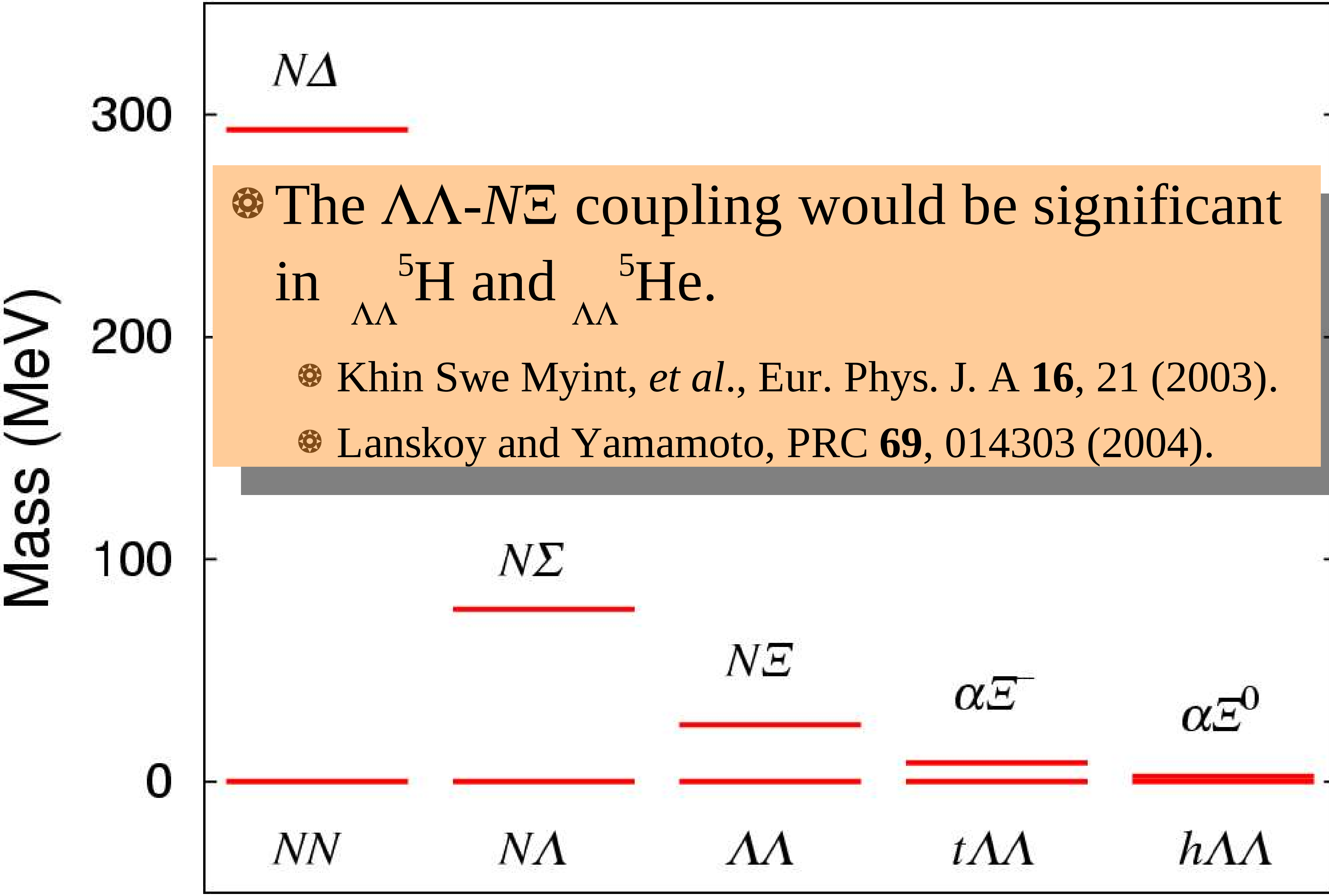
- Ne

A pass to S=-2 study

- There is no way to S=–2 study on the way avoinding ${}_{\Lambda}^{5}$He anomaly.



Exotic baryon admixture



- The $\Lambda\Lambda$-$N\Xi$ coupling would be significant in ${}_{\Lambda\Lambda}^{5}$H and ${}_{\Lambda\Lambda}^{5}$He.
  - Khin Swe Myint, *et al.*, Eur. Phys. J. A **16**, 21 (2003).
  - Lanskoy and Yamamoto, PRC **69**, 014303 (2004).

The purpose of this work

- Systematic study for the complete set of *s*-shell Λ hypernuclei with the strangeness $S=-1$ and -2 in a framework of full-coupled channel formulation.
- Theoretical search for ${}^{4}_{\Lambda\Lambda}\mathrm{H}$.
- Fully baryon mixing of ${}^{5}_{\Lambda\Lambda}\mathrm{H}$ and ${}^{5}_{\Lambda\Lambda}\mathrm{He}$.

- ${}^{3}_{\Lambda}\mathrm{H}$ $\sim NN\Lambda + NN\Sigma$
- ${}^{4}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{He}$ $\sim NNN\Lambda + NNN\Sigma$
- ${}^{5}_{\Lambda}\mathrm{He}$ $\sim NNNN\Lambda + NNNN\Sigma$
- ${}^{4}_{\Lambda\Lambda}\mathrm{H}$ $\sim NN\Lambda\Lambda + NN\Lambda\Sigma + NNN\Xi + NN\Sigma\Sigma$
- ${}^{5}_{\Lambda\Lambda}\mathrm{H}$, ${}^{5}_{\Lambda\Lambda}\mathrm{He}$ $\sim NNN\Lambda\Lambda + NNN\Lambda\Sigma + NNNN\Xi + NNN\Sigma\Sigma$
- ${}^{6}_{\Lambda\Lambda}\mathrm{He}$ $\sim NNNN\Lambda\Lambda + NNNN\Lambda\Sigma + NNNNN\Xi + NNNN\Sigma\Sigma$

NN, YN and YY potentials

- *NN* interaction: Minnesota potential
  - The *NN* interaction reproduces the low energy *NN* scattering data, and also reproduces reasonably well both the binding energies and sizes of ^{2}H, ^{3}H, ^{3}He, and ^{4}He.
- *YN* interaction: D2' potential
  - The *YN* interaction reproduces the experimental B_{Λ} of A=3-5 hypernuclei; Free from the $_{\Lambda}^{5}$He anomaly.
- *YY* interaction: Simulating Nijmegen model (ND or NF)
  - Fully coupled channel;

hard-core radius

ND: r_c=(0.56, 0.45) fm

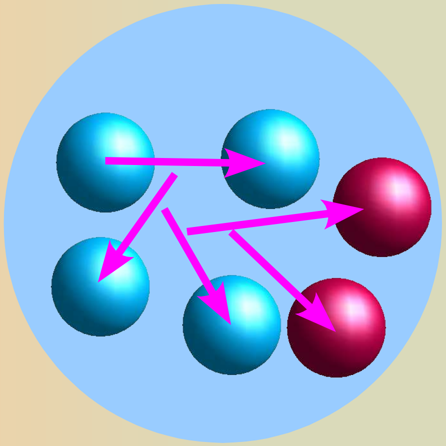
NF: (0.53, 0.52) fm

	$^{1}S_{0}$	$^{3}S_{1}$
I=0	$\Lambda\Lambda$-$N\Xi$-$\Sigma\Sigma$	$N\Xi$
I=1	$N\Xi$-$\Lambda\Sigma$	$N\Xi$-$\Lambda\Sigma$-$\Sigma\Sigma$
I=2	$\Sigma\Sigma$	

Ab initio calculation with stochastic variational method

- The variational trial function must be flexible enough to incorporate both
  - Explicit Σ degrees of freedom and
  - Higher orbital angular momenta.
- $\Psi = \sum_i c_i \, \Phi_{JMTM_T}(x; \mathbf{A}_i, u_i)$
- $\Phi_{JMIM_I}(x, \mathbf{A}_i, u_i)$

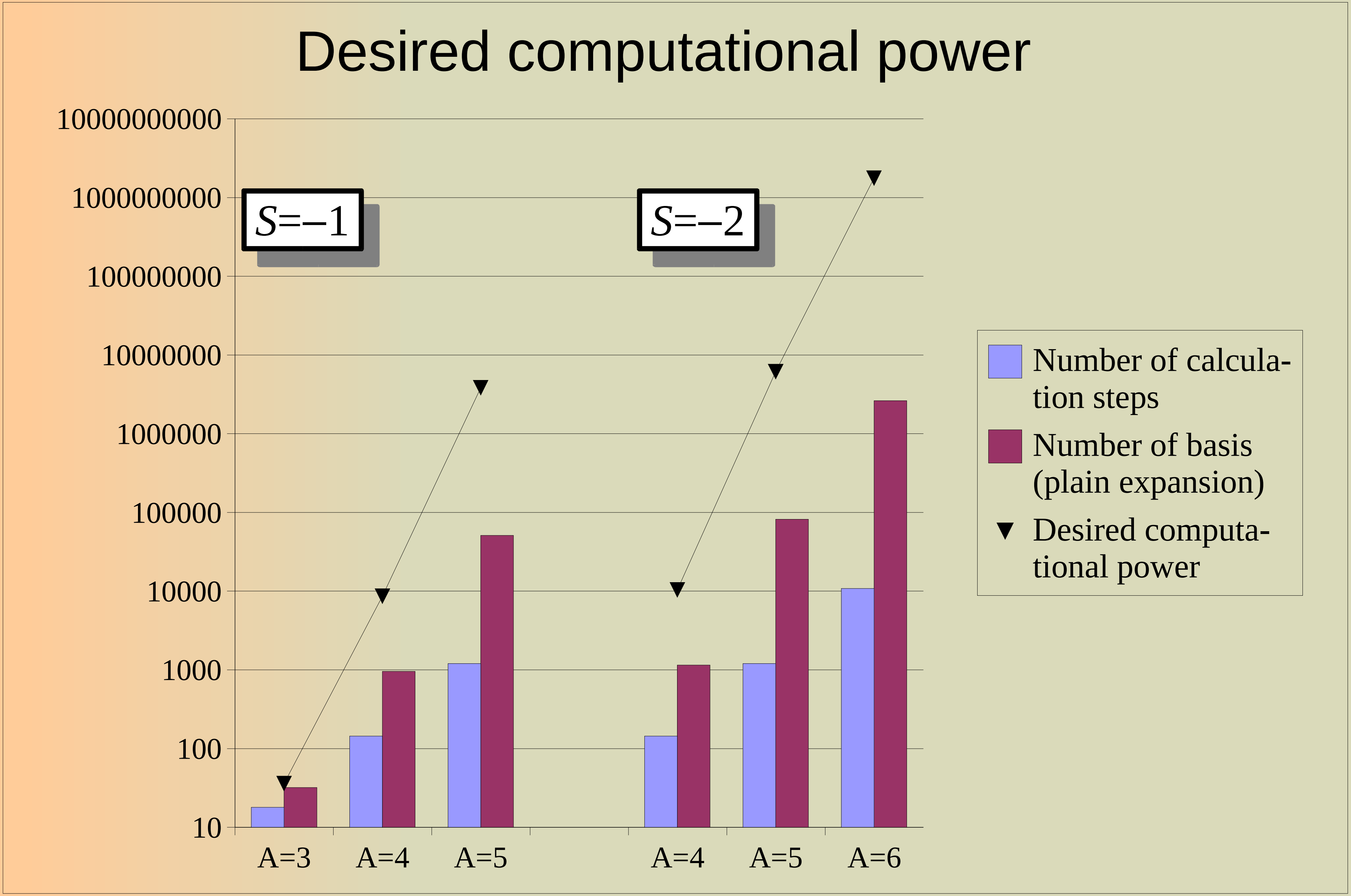
$$= \mathcal{A}\{G(x; \mathbf{A}_i)[\theta_{(kl)_i}(x; u_i) \times \chi_{s_i}]_{JM} \eta_{IM_I}\}$$



Complete six-body treatment

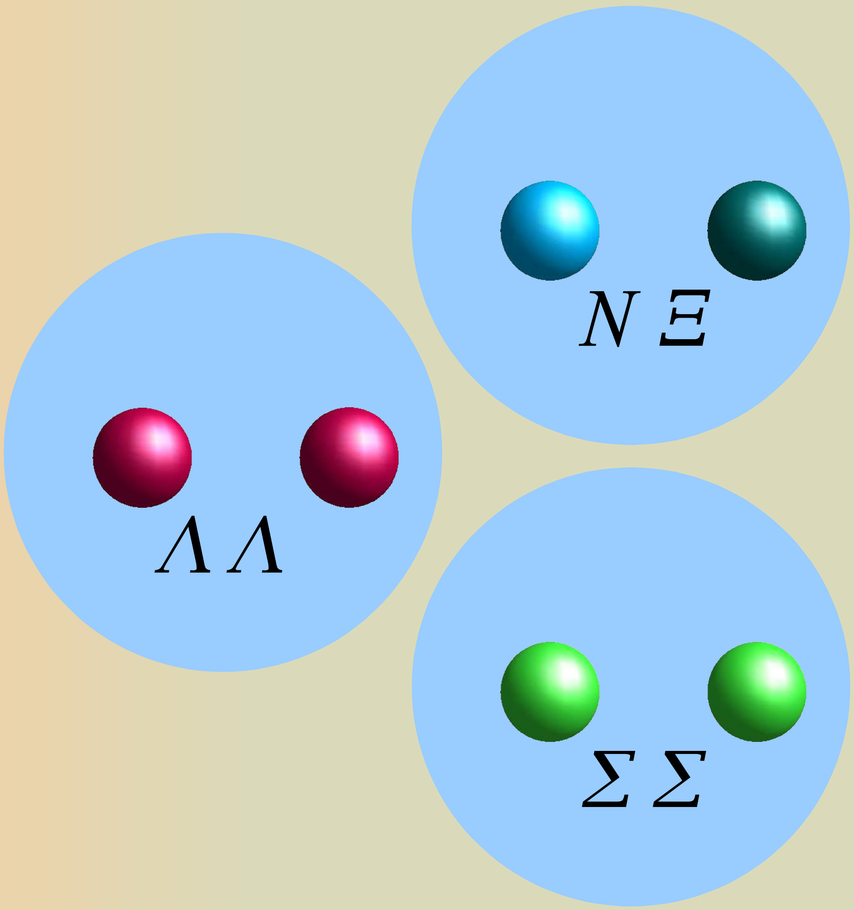
Ab initio calculation with SVM

- SVM is capable of handling the massive calculation.



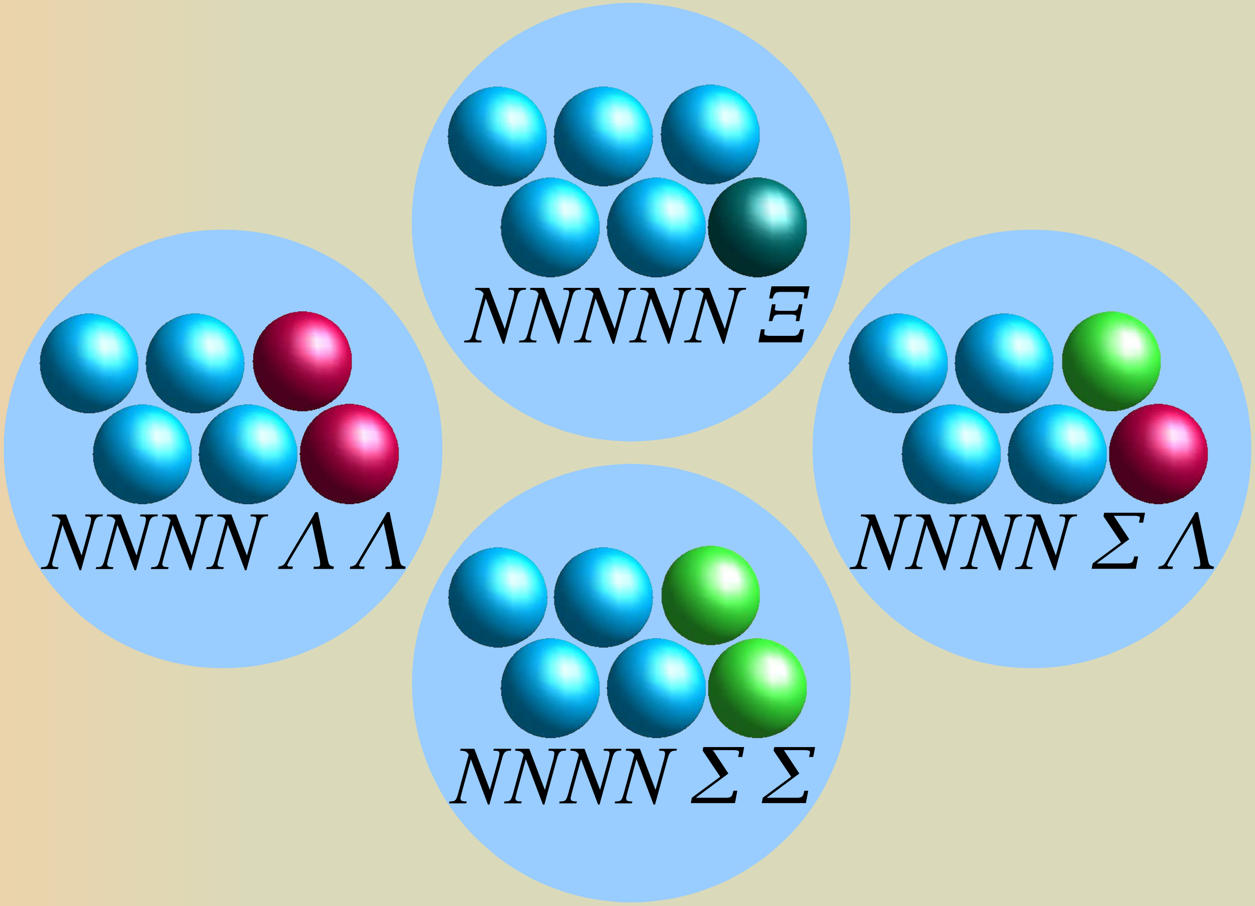
Ab initio calculation with stochastic variational method

- An example of isospin function



Ab initio calculation with stochastic variational method

- An example of isospin function
  - ${}_{\Lambda\Lambda}{}^{6}$He, ($J$=0, I=0); 16 channels



Ab initio calculation with stochastic variational method

- An example of isospin function
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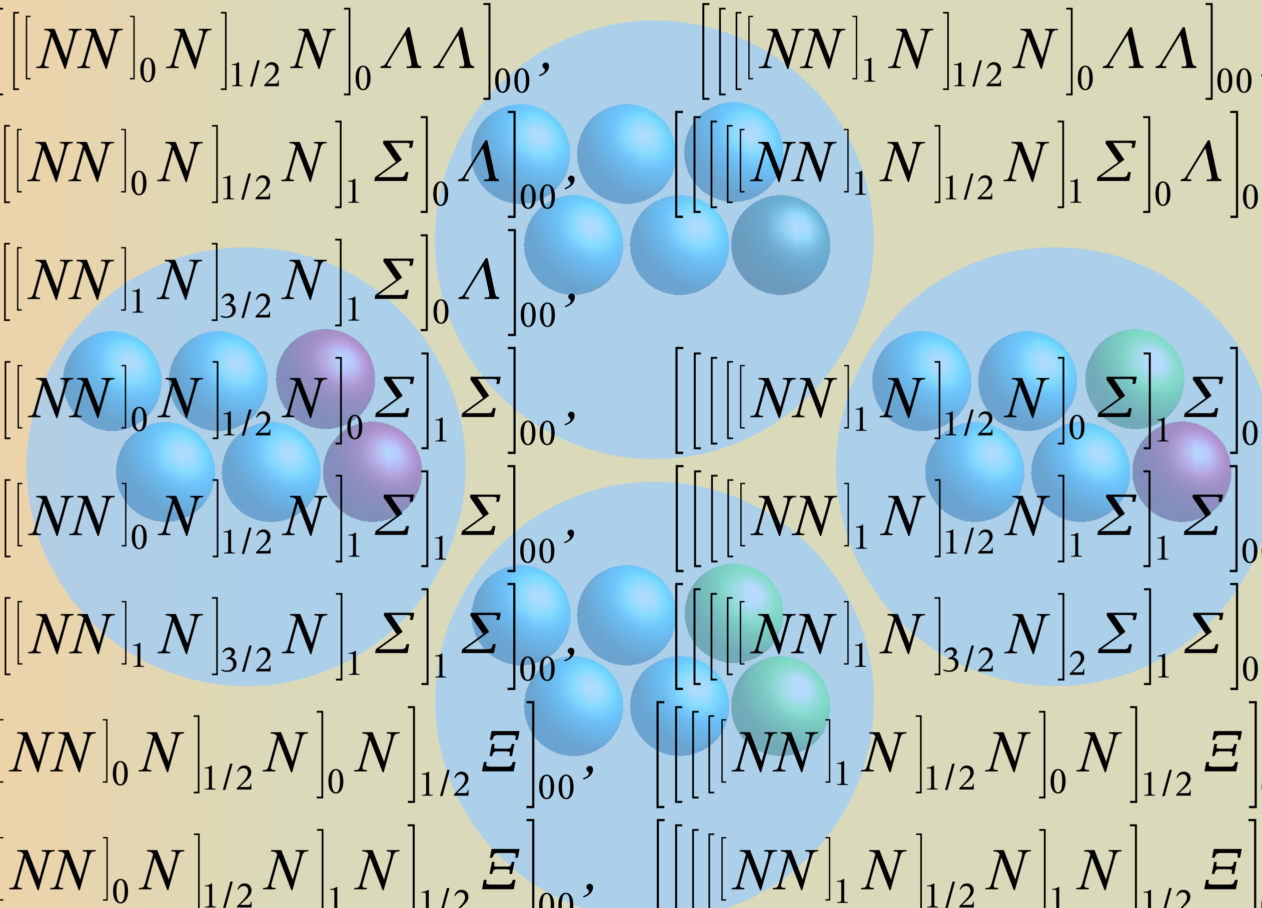
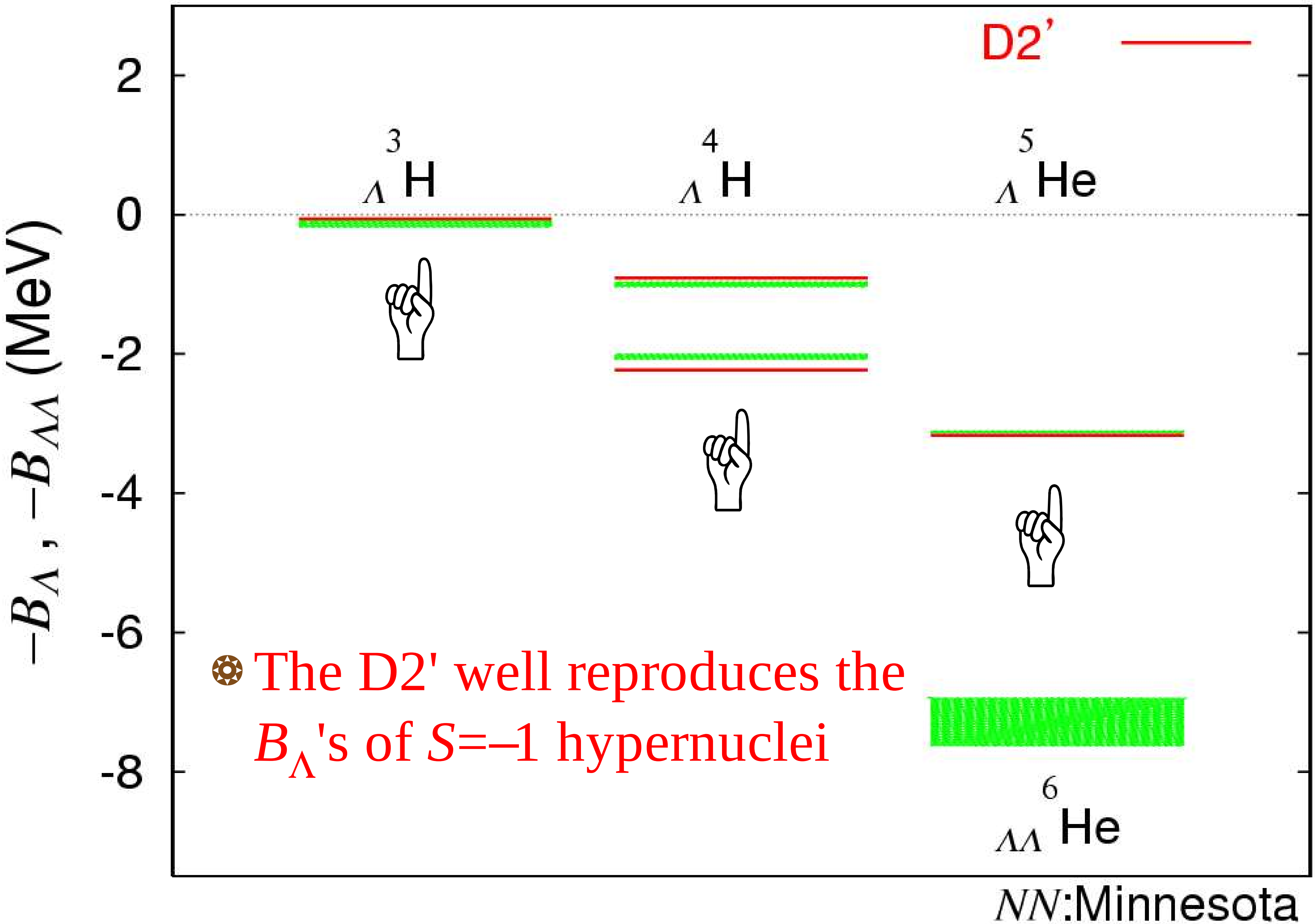


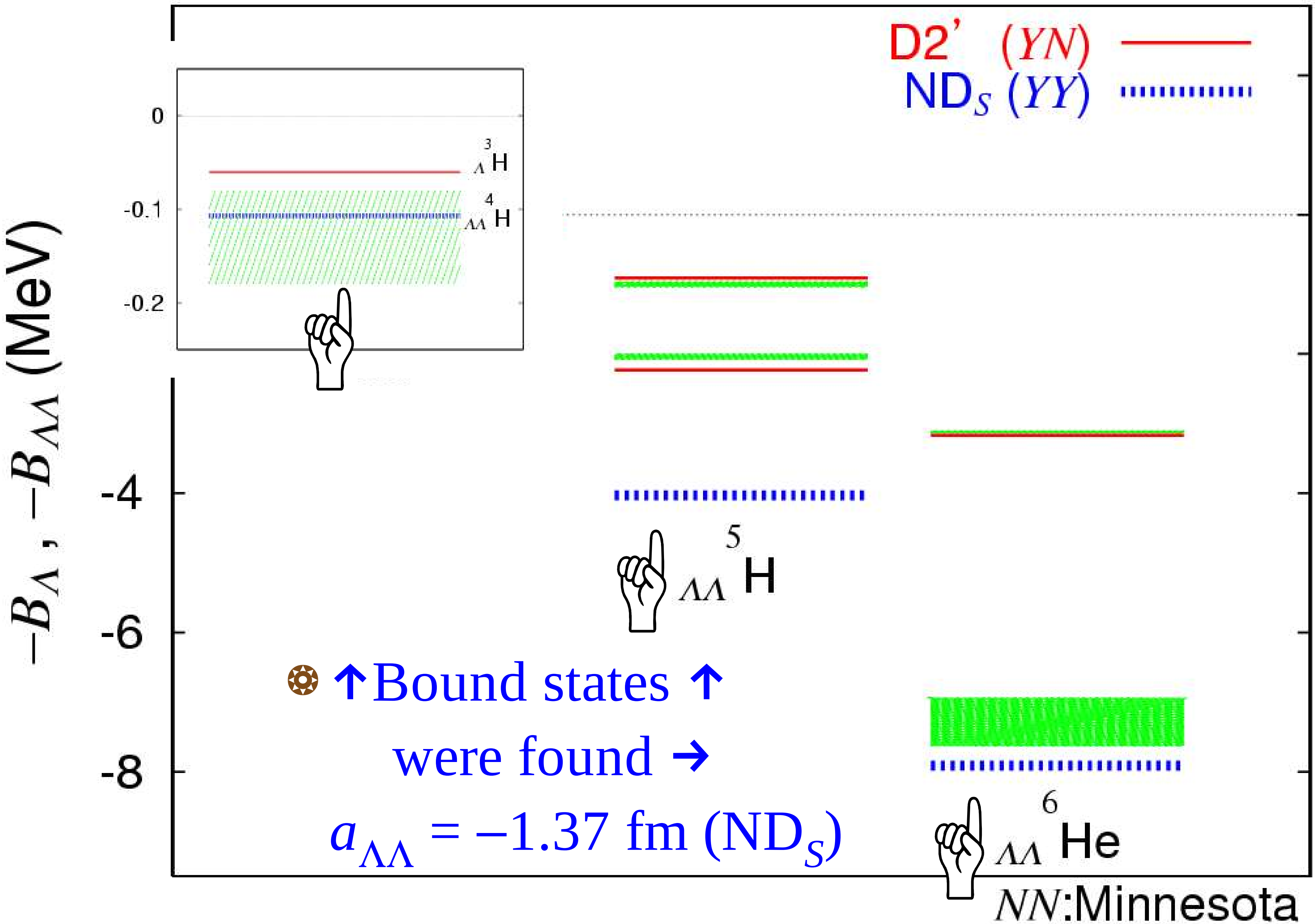
$$
\eta_{00} = \begin{aligned}
&\left[\left[\left[[NN]_0 N\right]_{1/2} N\right]_0 \Lambda\Lambda\right]_{00}, \quad \left[\left[\left[[NN]_1 N\right]_{1/2} N\right]_0 \Lambda\Lambda\right]_{00}, \\
&\left[\left[\left[\left[[NN]_0 N\right]_{1/2} N\right]_1 \Sigma\right]_0 \Lambda\right]_{00}, \quad \left[\left[\left[\left[[NN]_1 N\right]_{1/2} N\right]_1 \Sigma\right]_0 \Lambda\right]_{00}, \\
&\left[\left[\left[\left[[NN]_1 N\right]_{3/2} N\right]_1 \Sigma\right]_0 \Lambda\right]_{00}, \\
&\left[\left[\left[\left[[NN]_0 N\right]_{1/2} N\right]_0 \Sigma\right]_1 \Sigma\right]_{00}, \quad \left[\left[\left[\left[[NN]_1 N\right]_{1/2} N\right]_0 \Sigma\right]_1 \Sigma\right]_{00}, \\
&\left[\left[\left[\left[[NN]_0 N\right]_{1/2} N\right]_1 \Sigma\right]_1 \Sigma\right]_{00}, \quad \left[\left[\left[\left[[NN]_1 N\right]_{1/2} N\right]_1 \Sigma\right]_1 \Sigma\right]_{00}, \\
&\left[\left[\left[\left[[NN]_1 N\right]_{3/2} N\right]_1 \Sigma\right]_1 \Sigma\right]_{00}, \quad \left[\left[\left[\left[[NN]_1 N\right]_{3/2} N\right]_2 \Sigma\right]_1 \Sigma\right]_{00}, \\
&\left[\left[\left[\left[[NN]_0 N\right]_{1/2} N\right]_0 N\right]_{1/2} \Xi\right]_{00}, \quad \left[\left[\left[\left[[NN]_1 N\right]_{1/2} N\right]_0 N\right]_{1/2} \Xi\right]_{00}, \\
&\left[\left[\left[\left[[NN]_0 N\right]_{1/2} N\right]_1 N\right]_{1/2} \Xi\right]_{00}, \quad \left[\left[\left[\left[[NN]_1 N\right]_{1/2} N\right]_1 N\right]_{1/2} \Xi\right]_{00}, \\
&\left[\left[\left[\left[[NN]_1 N\right]_{3/2} N\right]_1 N\right]_{1/2} \Xi\right]_{00},
\end{aligned}
$$

Results



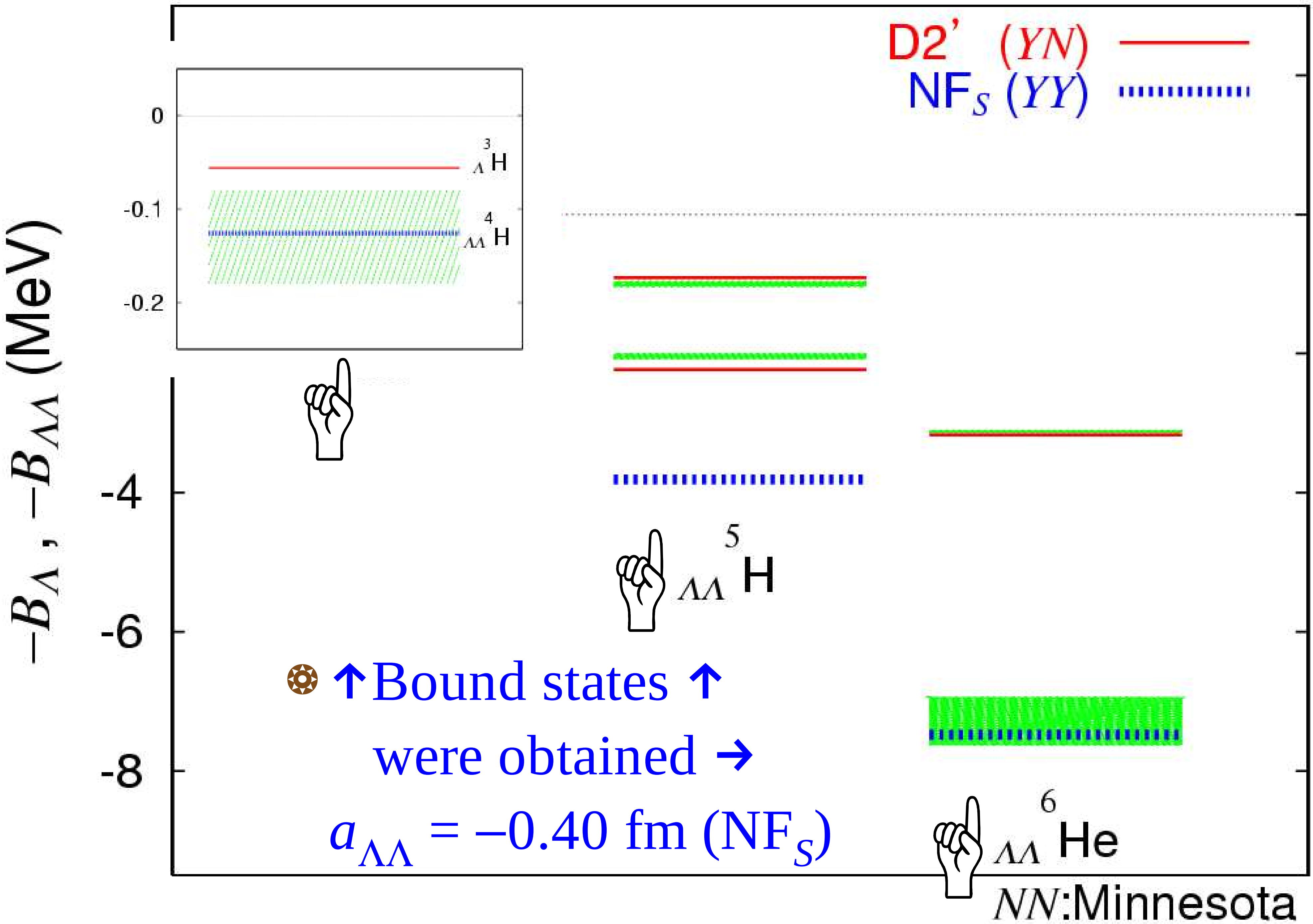
- The D2' well reproduces the B_{Λ}'s of S=–1 hypernuclei

Results



↑Bound states ↑
were found →
$a_{\Lambda\Lambda} = -1.37$ fm (ND_S)

Results



D2' (YN)
NF_S (YY)
−B_Λ , −B_ΛΛ (MeV)
0
-0.1
-0.2
3 Λ H
4 ΛΛ H
-4
-6
-8
5 ΛΛ H
6 ΛΛ He
↑Bound states ↑
were obtained →
a_ΛΛ = –0.40 fm (NF_S)
NN:Minnesota

Results

- Using the ND_S YY potential, we obtain

$$\Delta B_{\Lambda\Lambda}^{(\mathrm{calc})}({}_{\Lambda\Lambda}{}^{6}\mathrm{He}) = B_{\Lambda\Lambda}^{(\mathrm{calc})}({}_{\Lambda\Lambda}{}^{6}\mathrm{He}) - 2\,B_{\Lambda}^{(\mathrm{calc})}({}_{\Lambda}{}^{5}\mathrm{He})$$

$$= 1.55\ \mathrm{MeV},$$

which is slightly larger than the experimental value,

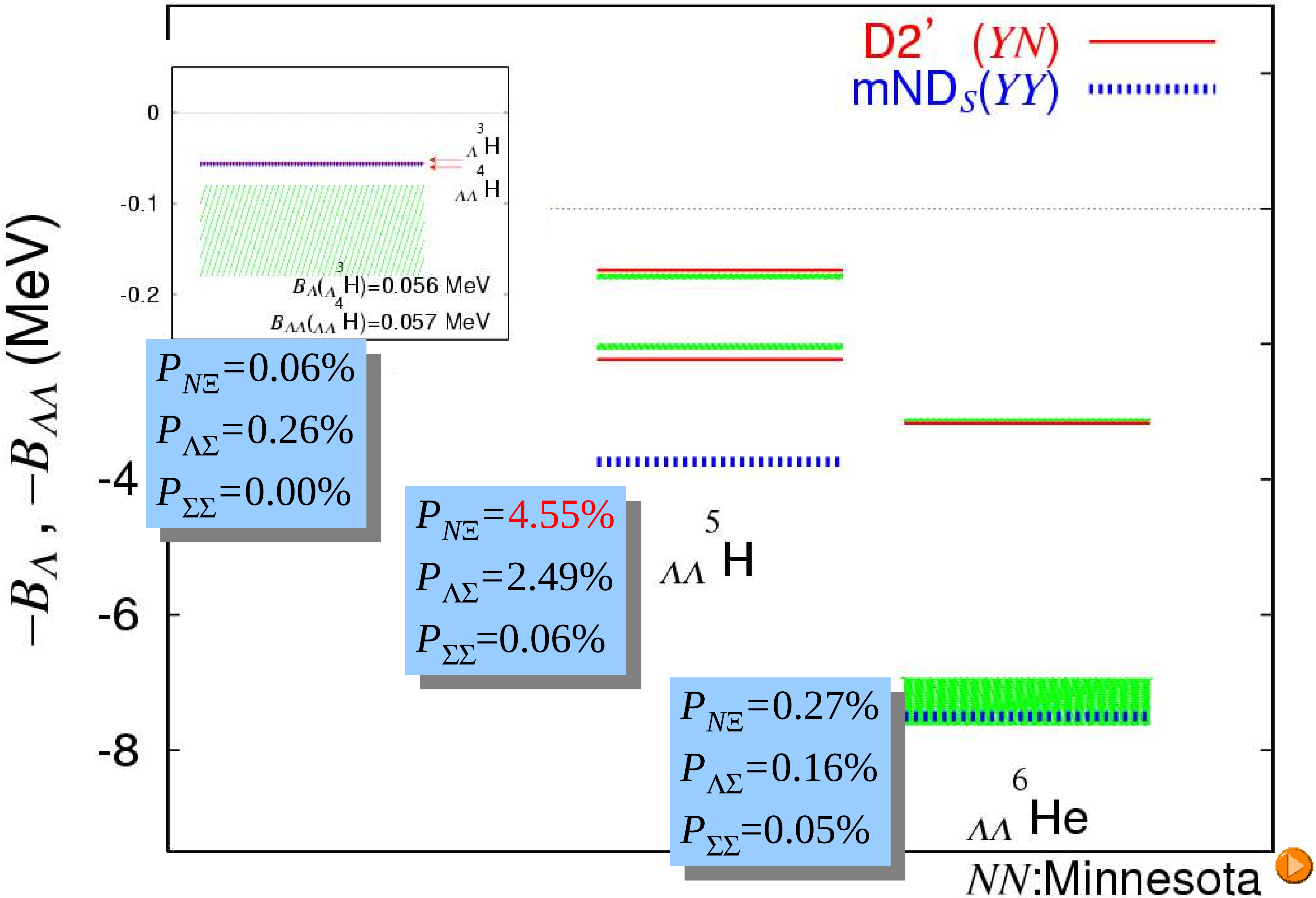
$$\Delta B_{\Lambda\Lambda}^{(\mathrm{exp})}({}_{\Lambda\Lambda}{}^{6}\mathrm{He}) = 1.01 \pm 0.20\ {}^{+0.18}_{-0.11}\ \mathrm{MeV}.$$

- On the other hand, using NF_S YY potential,

$$\Delta B_{\Lambda\Lambda}^{(\mathrm{calc})}({}_{\Lambda\Lambda}{}^{6}\mathrm{He}) = 1.12\ \mathrm{MeV},$$

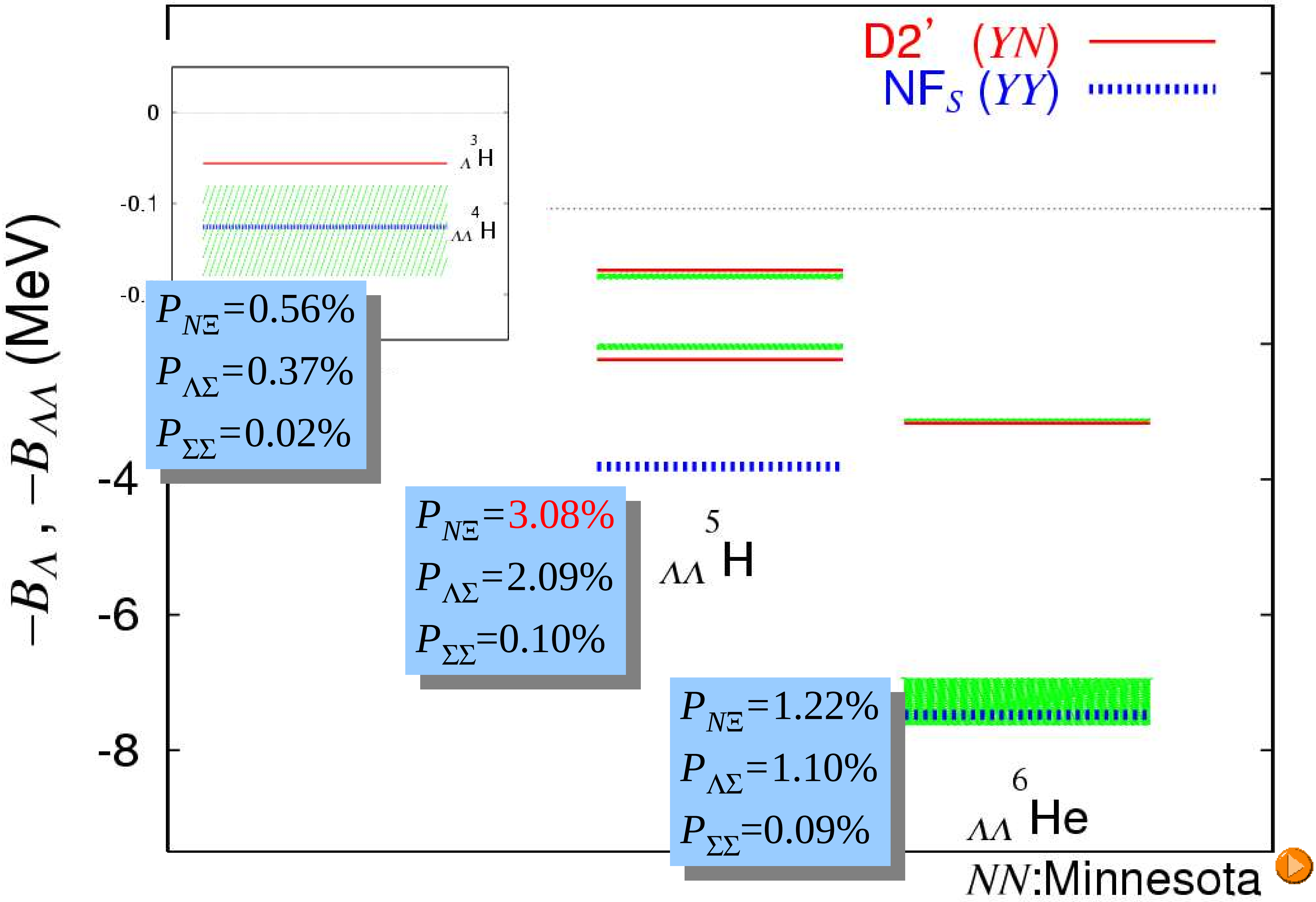
is fairly in good agreement with the experiment.
- We have also calculated the hypernuclei using a modified ND_S (mND_S) potential; the strength of the $\Lambda\Lambda$ diagonal part of the mND_S is reduced by multiplying by a factor 0.8, in order to reproduce the $\Delta B_{\Lambda\Lambda}^{(\mathrm{exp})}({}_{\Lambda\Lambda}{}^{6}\mathrm{He})$

Results



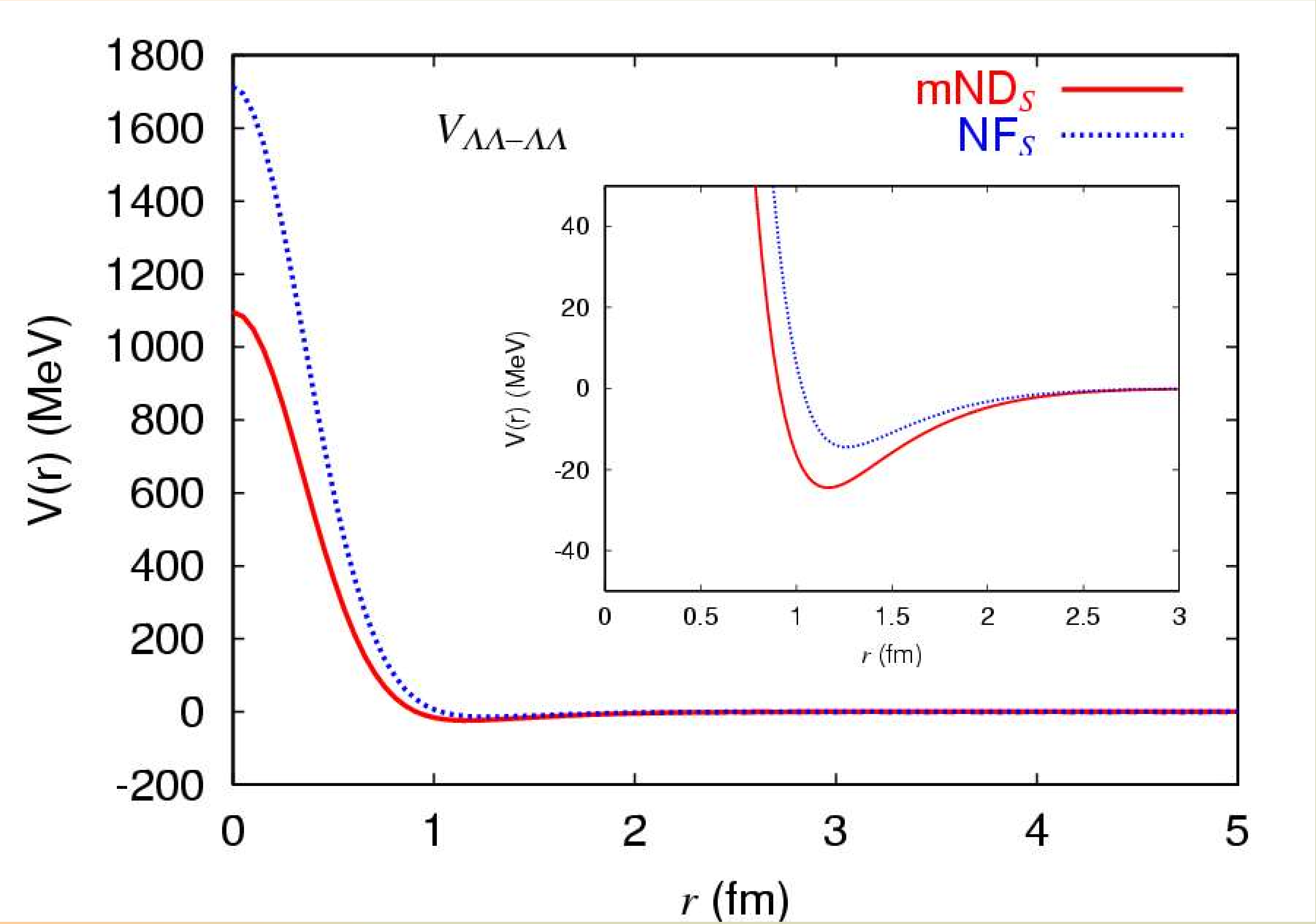
D2' (YN)
mND_S(YY)
0
-0.1
-0.2
3 Λ H
4 ΛΛ H
B_Λ(3 Λ H)=0.056 MeV
B_ΛΛ(4 ΛΛ H)=0.057 MeV
-4
-6
-8
−B_Λ , −B_ΛΛ (MeV)
P_NΞ=0.06%
P_ΛΣ=0.26%
P_ΣΣ=0.00%
P_NΞ=4.55%
P_ΛΣ=2.49%
P_ΣΣ=0.06%
5 ΛΛ H
P_NΞ=0.27%
P_ΛΣ=0.16%
P_ΣΣ=0.05%
6 ΛΛ He
NN:Minnesota

Results


D2' (YN)
NF_S (YY)
$-B_{\Lambda}$, $-B_{\Lambda\Lambda}$ (MeV)
0
-0.1
$^{3}_{\Lambda}$H
$^{4}_{\Lambda\Lambda}$H
-4
-6
-8
$P_{N\Xi}$=0.56%
$P_{\Lambda\Sigma}$=0.37%
$P_{\Sigma\Sigma}$=0.02%
$P_{N\Xi}$=3.08%
$P_{\Lambda\Sigma}$=2.09%
$P_{\Sigma\Sigma}$=0.10%
$^{5}_{\Lambda\Lambda}$H
$P_{N\Xi}$=1.22%
$P_{\Lambda\Sigma}$=1.10%
$P_{\Sigma\Sigma}$=0.09%
$^{6}_{\Lambda\Lambda}$He
NN:Minnesota

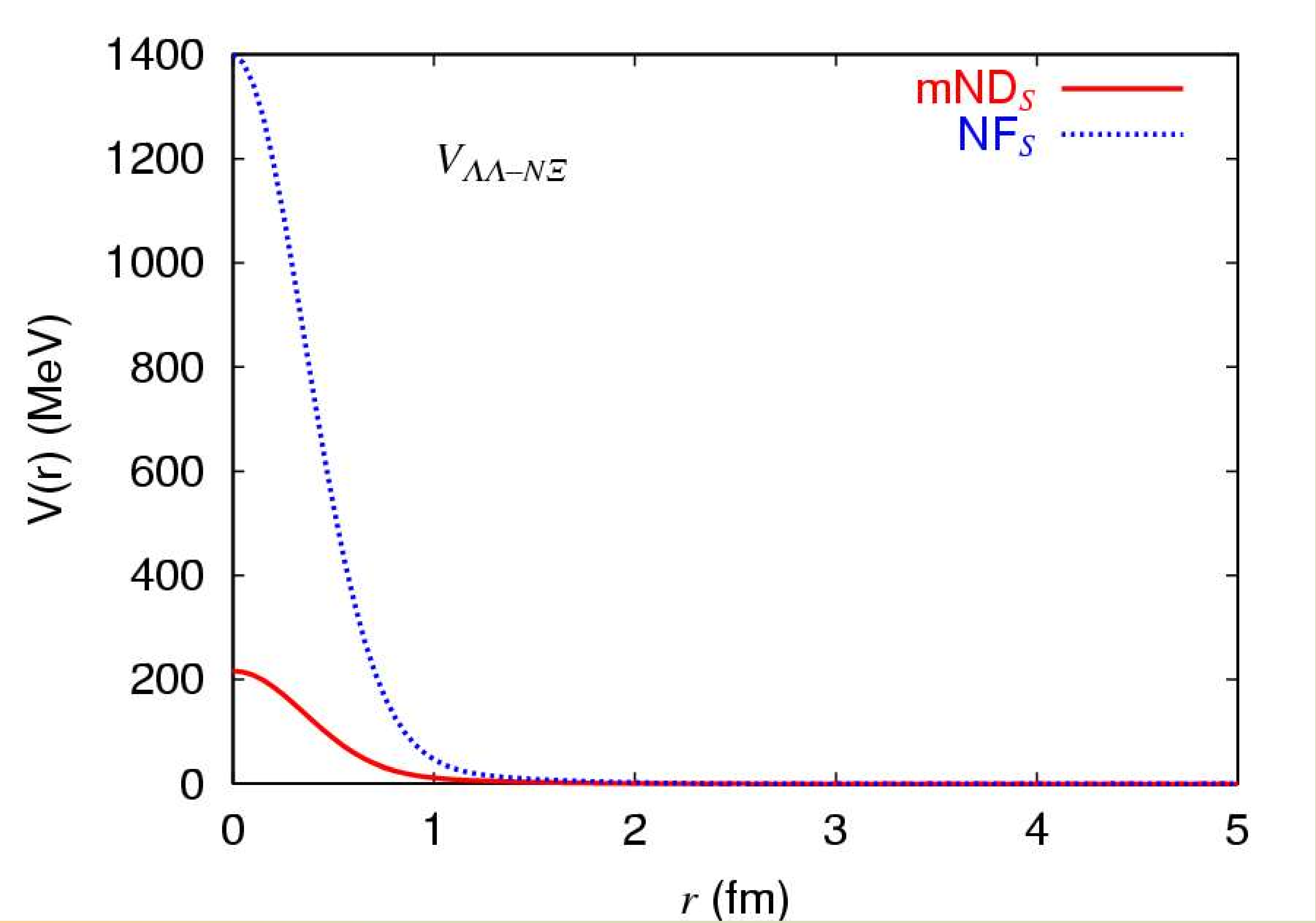
YY potentials

- The $V_{\Lambda\Lambda\text{-}\Lambda\Lambda}$ potential:



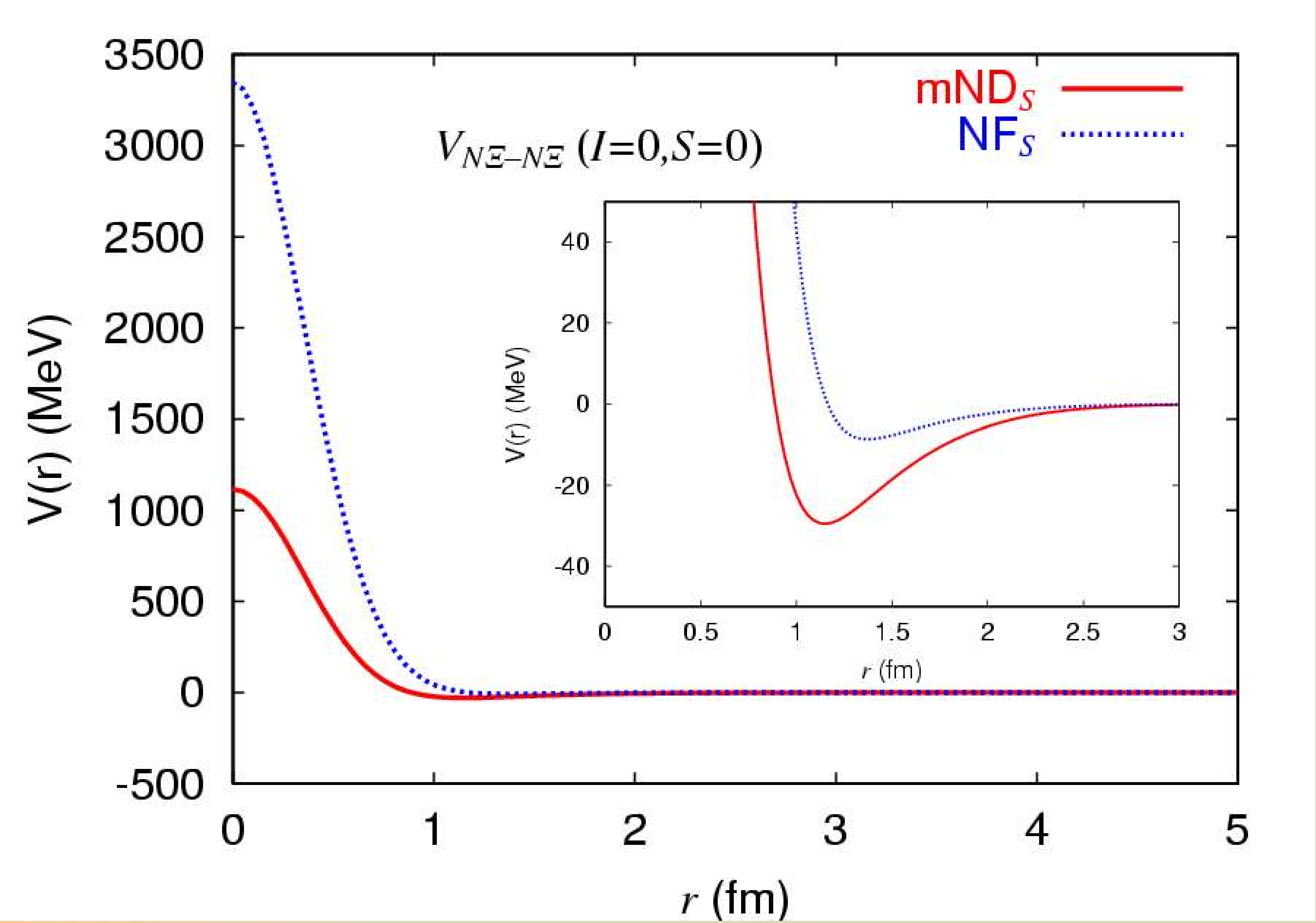
YY potentials

- The $V_{\Lambda\Lambda\text{-}N\Xi}$ potential:



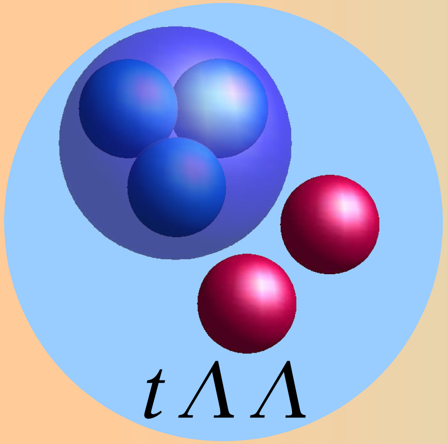
YY potentials

- The $V_{N\Xi\text{-}N\Xi}$ potential:



Baryon mixing in $_{\Lambda\Lambda}{}^{5}\mathrm{H}$

- We assume that all of the baryons occupy same (0*s*) orbit.

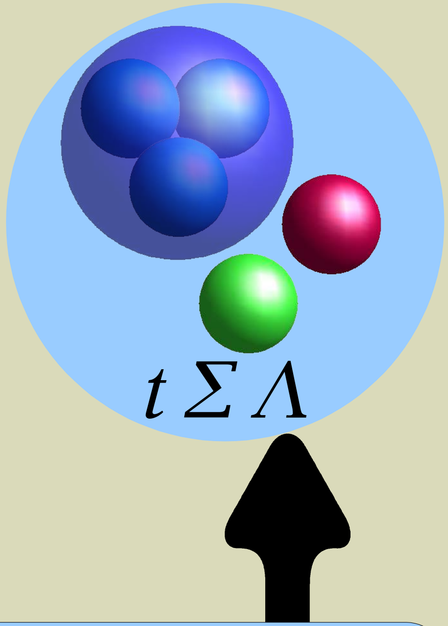


$$H=\begin{pmatrix} H_{\Lambda\Lambda} & V_{N\Xi-\Lambda\Lambda} & V_{\Lambda\Sigma-\Lambda\Lambda} \\ V_{\Lambda\Lambda-N\Xi} & H_{N\Xi} & V_{\Lambda\Sigma-N\Xi} \\ V_{\Lambda\Lambda-\Lambda\Sigma} & V_{N\Xi-\Lambda\Sigma} & H_{\Lambda\Sigma} \end{pmatrix},$$

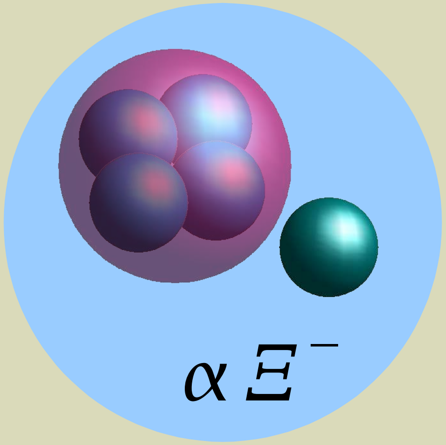
$$V_{\Lambda\Lambda-N\Xi}=v_{\Lambda\Lambda-N\Xi},$$

$$V_{\Lambda\Lambda-\Lambda\Sigma}=\sum_{i=1}^{N} v_{N_i\Lambda-N_i\Sigma}$$

$$V_{N\Xi-\Lambda\Sigma}=v_{N\Xi-\Lambda\Sigma}$$

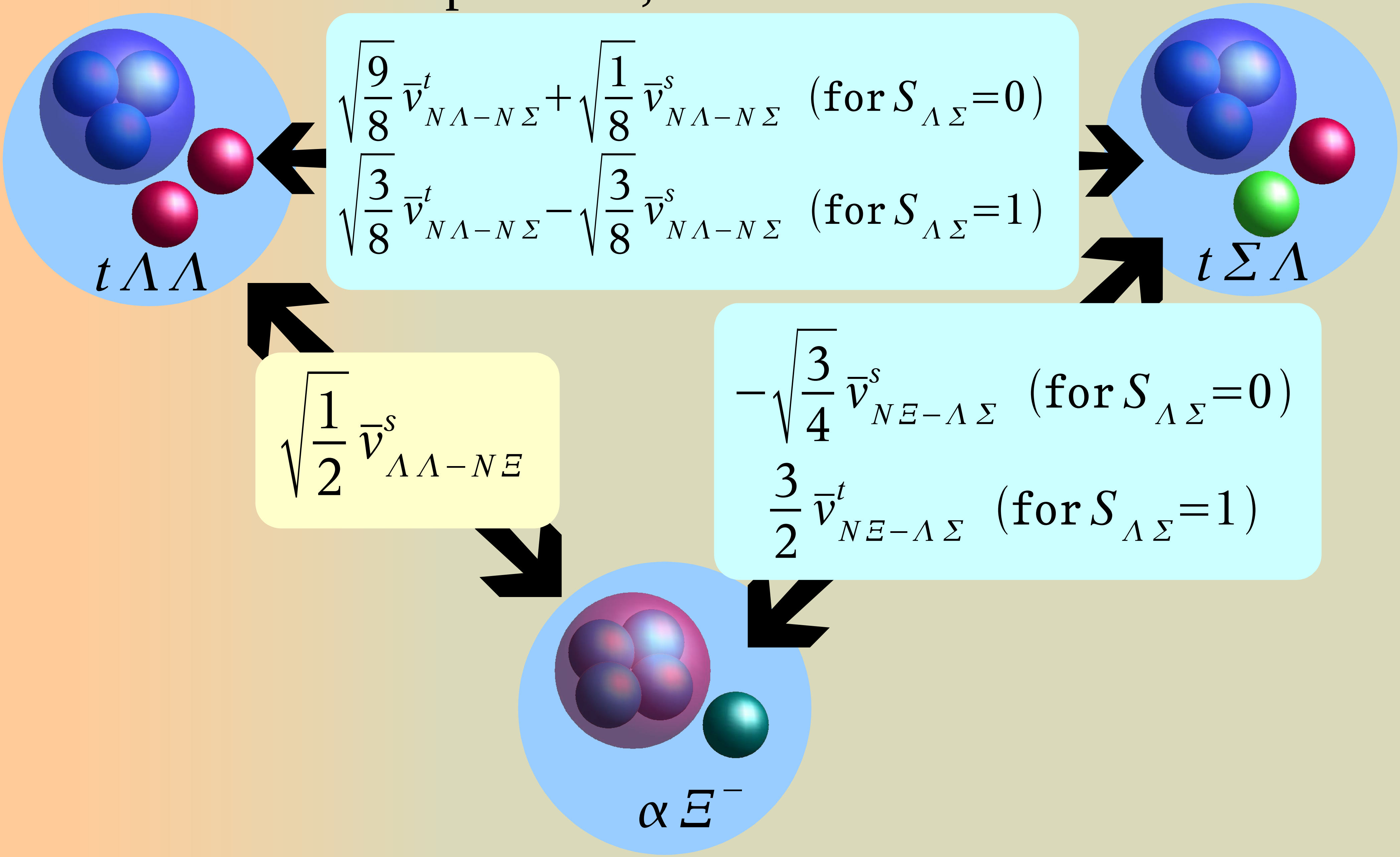


$$\Psi\left({}_{\Lambda\Sigma}^{5}\mathrm{H}\right).$$
$$=\sqrt{\frac{1}{3}}\left[\psi_t\times\left[\psi_{\Lambda\Sigma^0}\right]_{S_{\Lambda\Sigma}}\right]\times\psi_{\Lambda\Sigma^0-t}$$
$$-\sqrt{\frac{2}{3}}\left[\psi_h\times\left[\psi_{\Lambda\Sigma^-}\right]_{S_{\Lambda\Sigma}}\right]\times\psi_{\Lambda\Sigma^--h}$$
$$(\text{for } S_{\Lambda\Sigma}=0 \text{ or } 1)$$



Baryon mixing in $_{\Lambda\Lambda}{}^{5}$H

- Algebraic factors for each averaged coupling potential of the allowed spin state, v^s or v^t:



Baryon mixing in $_{\Lambda\Lambda}{}^{5}$H

- Solving the eigenvalue problem,

  $\det|h - \lambda E| = 0$,

  we have the

  ground state energy,

  $E = -11.82$ MeV

  (for the mND$_S$) or

  $E = -11.82$ MeV

  (for the NF$_S$), and

  $N\Xi$ probability,

  $P_{N\Xi} = 3.98$ %

  (for the mND$_S$) or

  $P_{N\Xi} = 2.83$ %

  (for the NF$_S$).

$$h = \begin{pmatrix} \frac{\langle H_{\Lambda\Lambda}\rangle}{P_{\Lambda\Lambda}} & \frac{\langle V_{N\Xi-\Lambda\Lambda}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{N\Xi}}} & \frac{\langle V_{\Lambda\Sigma-\Lambda\Lambda}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{\Lambda\Sigma}}} \\ \frac{\langle V_{\Lambda\Lambda-N\Xi}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{N\Xi}}} & \frac{\langle H_{N\Xi}\rangle}{P_{N\Xi}} & \frac{\langle V_{\Lambda\Sigma-N\Xi}\rangle}{\sqrt{P_{N\Xi}P_{\Lambda\Sigma}}} \\ \frac{\langle V_{\Lambda\Lambda-\Lambda\Sigma}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{\Lambda\Sigma}}} & \frac{\langle V_{N\Xi-\Lambda\Sigma}\rangle}{\sqrt{P_{N\Xi}P_{\Lambda\Sigma}}} & \frac{\langle H_{\Lambda\Sigma}\rangle}{P_{\Lambda\Sigma}} \end{pmatrix},$$

$$= \begin{pmatrix} -9.12 & -1.82 & -14.52 \\ -1.82 & 5.02 & -10.37 \\ -14.52 & -10.37 & 92.45 \end{pmatrix} \quad (\text{for the mND}_S),$$

or

$$= \begin{pmatrix} -6.10 & -20.47 & -14.91 \\ -20.47 & 115.3 & -10.01 \\ -14.91 & -10.01 & 101.6 \end{pmatrix} \quad (\text{for the NF}_S).$$

Baryon mixing in ${}_{\Lambda\Lambda}{}^{5}\mathrm{H}$

- Solving the eigenvalue problem of only the first 2x2 subspace,

  $\det | h - \lambda E | = 0$,

  we have the

  ground state energy,

  $E = -9.35$ MeV

  (for the mND_S) or

  $E = -9.46$ MeV

  (for the NF_S), and

  $N\Xi$ probability,

  $P_{N\Xi} = 1.57$ %

  (for the mND_S) or

  $P_{N\Xi} = 2.62$ %

  (for the NF_S).

$$h = \begin{pmatrix} \frac{\langle H_{\Lambda\Lambda}\rangle}{P_{\Lambda\Lambda}} & \frac{\langle V_{N\Xi-\Lambda\Lambda}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{N\Xi}}} & \frac{\langle V_{\Lambda\Sigma-\Lambda\Lambda}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{\Lambda\Sigma}}} \\ \frac{\langle V_{\Lambda\Lambda-N\Xi}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{N\Xi}}} & \frac{\langle H_{N\Xi}\rangle}{P_{N\Xi}} & \frac{\langle V_{\Lambda\Sigma-N\Xi}\rangle}{\sqrt{P_{N\Xi}P_{\Lambda\Sigma}}} \\ \frac{\langle V_{\Lambda\Lambda-\Lambda\Sigma}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{\Lambda\Sigma}}} & \frac{\langle V_{N\Xi-\Lambda\Sigma}\rangle}{\sqrt{P_{N\Xi}P_{\Lambda\Sigma}}} & \frac{\langle H_{\Lambda\Sigma}\rangle}{P_{\Lambda\Sigma}} \end{pmatrix},$$

$$= \begin{pmatrix} -9.12 & -1.82 & -14.52 \\ -1.82 & 5.02 & -10.37 \\ -14.52 & -10.37 & 92.45 \end{pmatrix} \quad (\text{for the mND}_S),$$

or

$$= \begin{pmatrix} -6.10 & -20.47 & -14.91 \\ -20.47 & 115.3 & -10.01 \\ -14.91 & -10.01 & 101.6 \end{pmatrix} \quad (\text{for the NF}_S).$$

Baryon mixing in $_{\Lambda\Lambda}{}^{5}$H

- The $N\Lambda$-$N\Sigma$ and $N\Xi$-$\Lambda\Sigma$ potentials enhance the ground state energy,

  $E = -9.35$ MeV

  ⇊

  $E = -11.82$ MeV,

  and also the $N\Xi$ probability,

  $P_{N\Xi} = 1.57$ %

  ⇊

  $P_{N\Xi} = 3.98$ %,

  for the mND$_S$ potential.

$$h = \begin{pmatrix} \frac{\langle H_{\Lambda\Lambda}\rangle}{P_{\Lambda\Lambda}} & \frac{\langle V_{N\Xi-\Lambda\Lambda}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{N\Xi}}} & \frac{\langle V_{\Lambda\Sigma-\Lambda\Lambda}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{\Lambda\Sigma}}} \\ \frac{\langle V_{\Lambda\Lambda-N\Xi}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{N\Xi}}} & \frac{\langle H_{N\Xi}\rangle}{P_{N\Xi}} & \frac{\langle V_{\Lambda\Sigma-N\Xi}\rangle}{\sqrt{P_{N\Xi}P_{\Lambda\Sigma}}} \\ \frac{\langle V_{\Lambda\Lambda-\Lambda\Sigma}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{\Lambda\Sigma}}} & \frac{\langle V_{N\Xi-\Lambda\Sigma}\rangle}{\sqrt{P_{N\Xi}P_{\Lambda\Sigma}}} & \frac{\langle H_{\Lambda\Sigma}\rangle}{P_{\Lambda\Sigma}} \end{pmatrix},$$

$$= \begin{pmatrix} -9.12 & -1.82 & -14.52 \\ -1.82 & 5.02 & -10.37 \\ -14.52 & -10.37 & 92.45 \end{pmatrix} \quad (\text{for the mND}_S),$$

or

$$= \begin{pmatrix} -6.10 & -20.47 & -14.91 \\ -20.47 & 115.3 & -10.01 \\ -14.91 & -10.01 & 101.6 \end{pmatrix} \quad (\text{for the NF}_S).$$

Baryon mixing in ${}_{\Lambda\Lambda}{}^{5}$H

- The $N\Lambda$-$N\Sigma$ and $N\Xi$-$\Lambda\Sigma$ potentials enhance the ground state energy,

  $E = -9.46$ MeV

  ⇊

  $E = -11.82$ MeV,

  but hardly enhance the $N\Xi$ probability,

  $P_{N\Xi} = 2.62$ %

  ⇊

  $P_{N\Xi} = 2.83$ %,

  for the NF$_S$ potential.

$$h = \begin{pmatrix} \frac{\langle H_{\Lambda\Lambda}\rangle}{P_{\Lambda\Lambda}} & \frac{\langle V_{N\Xi-\Lambda\Lambda}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{N\Xi}}} & \frac{\langle V_{\Lambda\Sigma-\Lambda\Lambda}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{\Lambda\Sigma}}} \\ \frac{\langle V_{\Lambda\Lambda-N\Xi}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{N\Xi}}} & \frac{\langle H_{N\Xi}\rangle}{P_{N\Xi}} & \frac{\langle V_{\Lambda\Sigma-N\Xi}\rangle}{\sqrt{P_{N\Xi}P_{\Lambda\Sigma}}} \\ \frac{\langle V_{\Lambda\Lambda-\Lambda\Sigma}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{\Lambda\Sigma}}} & \frac{\langle V_{N\Xi-\Lambda\Sigma}\rangle}{\sqrt{P_{N\Xi}P_{\Lambda\Sigma}}} & \frac{\langle H_{\Lambda\Sigma}\rangle}{P_{\Lambda\Sigma}} \end{pmatrix},$$

$$= \begin{pmatrix} -9.12 & -1.82 & -14.52 \\ -1.82 & 5.02 & -10.37 \\ -14.52 & -10.37 & 92.45 \end{pmatrix} \quad (\text{for the mND}_S),$$

or

$$= \begin{pmatrix} -6.10 & -20.47 & -14.91 \\ -20.47 & 115.3 & -10.01 \\ -14.91 & -10.01 & 101.6 \end{pmatrix} \quad (\text{for the NF}_S).$$

Summary

- We have performed a systematic study for S=–2 hypernuclei (${}^{4}_{\Lambda\Lambda}$H, ${}^{5}_{\Lambda\Lambda}$H, ${}^{5}_{\Lambda\Lambda}$He, ${}^{6}_{\Lambda\Lambda}$He), in a complete six-body and fully coupled channel treatment.
- Using a set of baryon-baryon potentials among the octet baryons which is consistent with all of the experimental binding energies of s-shell (hyper-)nuclei, ${}^{4}_{\Lambda\Lambda}$H has a particle stable bound state. ➜ ${}^{4}_{\Lambda\Lambda}$H could exist.
- Fully baryon mixing of the ${}^{5}_{\Lambda\Lambda}$H and ${}^{5}_{\Lambda\Lambda}$He.
  - Larger $P_{N\Xi}$ probability has been obtained even for the weaker ΛΛ-NΞ potential of the mND$_S$.
  - The ΛN-ΣN and ΞN-ΛΣ potential play important roles and significantly enhance the ground state en-

In the future study

- The present study is the first attempt to explore the few-body systems with multistrangeness in a fully coupled channel scheme.
- Further studies should be made:
  - Structure of ${}_{\Lambda\Lambda}^{4}H$.
  - Structure of ${}_{\Lambda\Lambda}^{5}H$ and ${}_{\Lambda\Lambda}^{5}He$.
    - Charge symmetry breaking between ${}_{\Lambda\Lambda}^{5}H$ and ${}_{\Lambda\Lambda}^{5}He$.
  - Structure of ${}_{\Lambda\Lambda}^{6}He$.
- The other exotic few-body systems with strangeness.

In the future study

- The other exotic few-body systems with strangeness.
  - Kaon-nucleus systems.
  - Exotic structure of baryon such as $\Lambda(1405)$.
  - Pentaquark.
- Relativistic effects should be taken into account.
  - Semi-relativistic hamiltonian in the center-of-mass system.

$$H = \sum_{i=1}^{A} \left(\sqrt{m_i^2 + p_i^2} - m_i\right) + \sum_{i<j} V_{ij}, \qquad \left(\text{with } \sum_{i=1}^{A} p_i = 0\right)$$

  - For example, we suppose the ^{3}H of $(0s)^3$, with size parameter b.

$$\phi = \exp\left\{-\frac{1}{2} b^2 \sum_{i=1}^{3} r_i^2\right\}$$