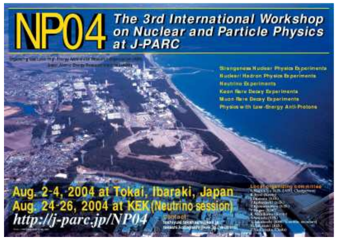
The s–wave πN Scattering Length

From pionic deuterium, hydrogen, and πN data, using a unitarized coupled channel approach

M. Döring, E. Oset, and M.J. Vicente Vacas

IFIC, University of Valencia, Spain

Affiliation



CSIC

nucl-th/0402086

NP04 conference, August 2–4 2004, Tokai, Japan

Outline I

- **Part I:**

  Corrections to the pion–deuteron scattering length $a_{\pi^- d}$

- Faddeev Equations for Re $a_{\pi d}$
- Absorption and Dispersion
- The $\Delta(1232)$ resonance, crossed diagrams, and others
- Fermi corrections

Outline II

- **Part II:**

  The unitary chiral model for low energy πN–scattering
- Summary of the theory
- Threshold constraints in the (b_0, b_1) plane
- Consistency of the model, threshold versus low energy scattering
- Parametrization of the πN potential
- Isospin breaking

Isoscalar and Isovector Scattering Lengths

$$
\begin{aligned}
b_0 &= \frac{1}{2}\left(a_{\pi^- n} + a_{\pi^- p}\right) \\
b_1 &= \frac{1}{2}\left(a_{\pi^- n} - a_{\pi^- p}\right)
\end{aligned}
$$

- Isospin symmetry: πN scattering is a function of (b_0, b_1) only. Averaged masses.

versus

- No assumption of isospin symmetry: Physical masses. The elementary scattering lengths $a_{\pi N}$ are independent of each other. Isospin breaking.

Experimental threshold values

$$a_{\pi^- p \to \pi^- p} = (883 \pm 2 \text{ stat. } \pm 10 \text{ sys. }) \cdot 10^{-4} m_\pi^{-1}$$

$$a_{\pi^- p \to \pi^0 n} = -1280(60) \cdot 10^{-4} m_\pi^{-1}$$

$$a_{\pi^- d} = (-252 \pm 5 \text{ stat. } \pm 5 \text{ sys. } + i\, 63(7)) \cdot 10^{-4} m_\pi^{-1}$$

Data: Schroeder *et al.* PLB 469, Sigg *et al.* NPA 609, Sigg *et al.* PRL 75, Schroeder *et al.* EPJC 21

The three body scattering problem requires an analysis of corrections to $a_{\pi^- d}$. With these modifications, the constraint from $a_{\pi^- d}$ can be used for an exact determination of b_0 and b_1.
The $\pi^- d$ scattering length is sensitive to b_0.

Multiple Scattering in the πd system

(b_0, b_1)	$(-0.0001, -0.0885)$	$(-0.0131, -0.0924)$
IA	$-2.14 \cdot 10^{-4}$	-0.02793
Double Scattering	-0.02527	-0.02725
Triple Scattering	0.002697	0.003489
4– and higher	$1.06 \cdot 10^{-4}$	$5.4 \cdot 10^{-5}$
Solution Faddeev	-0.02268	-0.05163
Exp. Re $a_{\pi^- d}$:	-0.0252 ± 10	-0.0252 ± 10

Units $[m_{\pi^-}^{-1}]$

Faddeev Equations

- Faddev equations:

$$
\begin{aligned}
T_p &= t_p + t_p G T_n + t_p^x G T_n^x \\
T_n &= t_n + t_n G T_p \\
T_n^x &= t_n^x + t_n^0 G T_n^x + t_n^x G T_n \\
T_{\pi^- d} &= T_p + T_n
\end{aligned}
$$

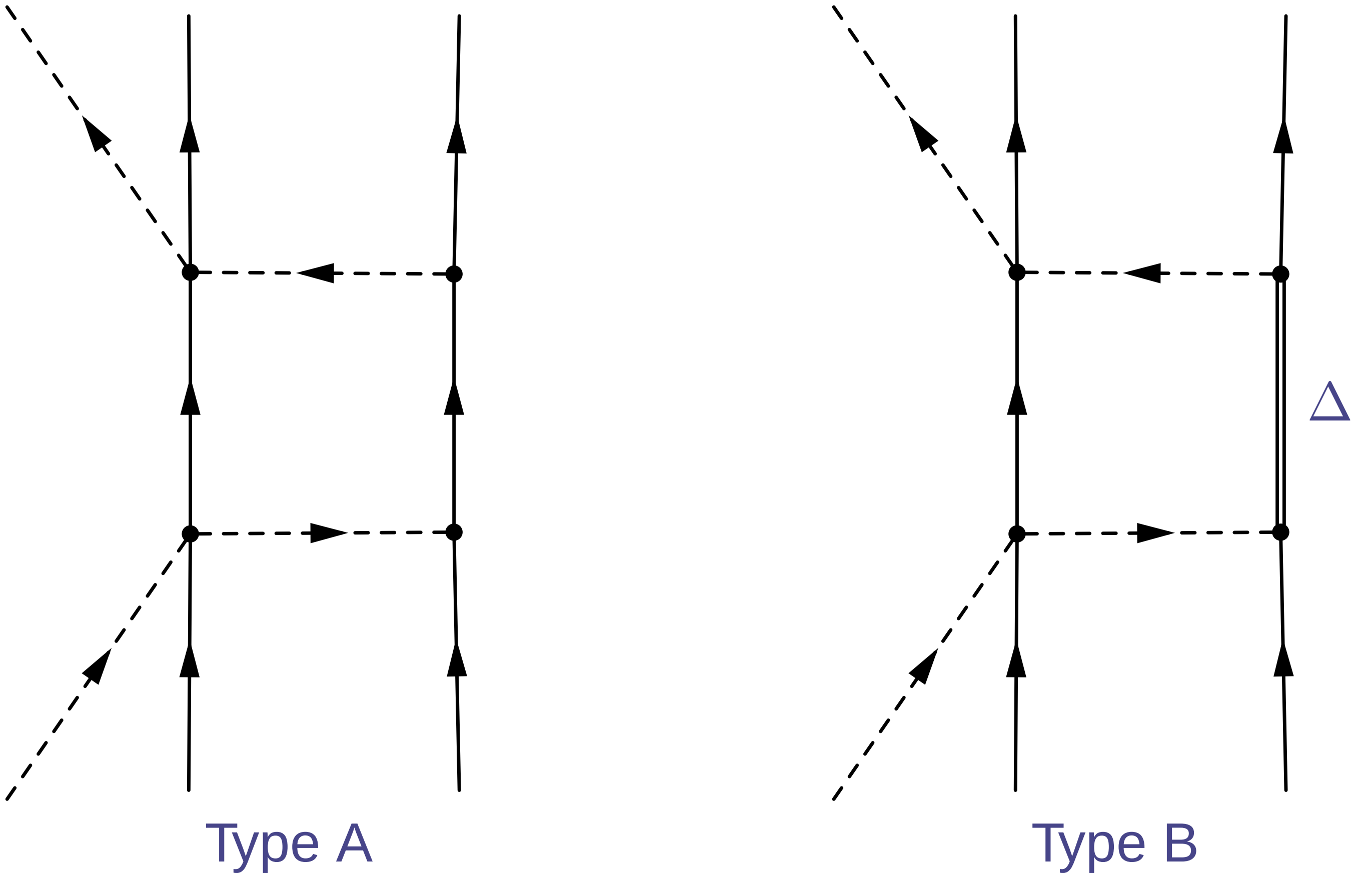
- Folding with the deuteron wave function:

$$a_{\pi^- d} = \frac{M_d}{m_{\pi^-} + M_d} \int d\mathbf{r}\ |\varphi_d(\mathbf{r})|^2 \hat{A}_{\pi^- d}(r)$$

- The Faddeev eqns. relate the elementary scattering lengths $a_{\pi N}$ to $a_{\pi^- d}$:

$$a_{\pi^- d} = f(a_{\pi^- p}, a_{\pi^- n}, a_{\pi^0 n}, a_{\pi^- p \to \pi^0 n})$$

Absorption and Dispersion



Absorption amplitude I

- Koltun–Reitan Hamiltonian:

$$H_I = 4\pi \left[\frac{\lambda_1}{\mu} \, \overline{\Psi} \vec{\phi} \vec{\phi} \Psi + \frac{\lambda_2}{\mu^2} \, \overline{\Psi} \vec{\tau} (\vec{\phi} \times \partial^0 \vec{\phi}) \Psi \right]$$

- Yukawa–type p–wave coupling:

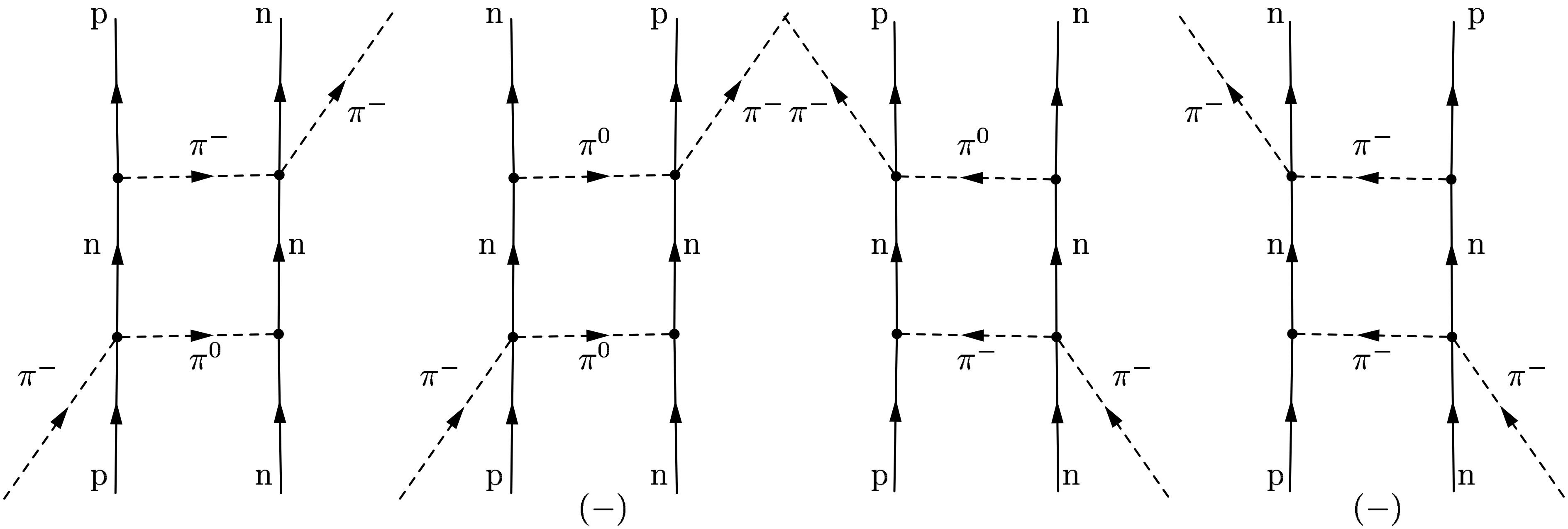
$$\vec{\sigma}\mathbf{q}' \, \vec{\sigma}\mathbf{q} = \mathbf{q}' \cdot \mathbf{q} + i \, (\mathbf{q}' \times \mathbf{q}) \, \vec{\sigma}$$

- Amplitude

$$\begin{aligned} T \quad = \quad & i \int \frac{d^4 l}{(2\pi^4)} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \int \frac{d^3\mathbf{q}'}{(2\pi)^3} \, F_d(\mathbf{q}+\mathbf{l}) F_d(\mathbf{q}'+\mathbf{l}) \\ & \frac{1}{q^2 - m_\pi^2 + i\epsilon} \, \frac{1}{q'^2 - m_\pi^2 + i\epsilon} \, \frac{1}{l^0 - \epsilon(\mathbf{l}) + i\epsilon} \, \frac{1}{l'^0 - \epsilon(\mathbf{l}') + i\epsilon} \\ & \Sigma t_1 \; t_2 \; t_{1'} \; t_{2'} \; \left(\vec{\sigma}\mathbf{q}' \, \vec{\sigma}\mathbf{q} \right) \end{aligned}$$

Absorption amplitude II

- Additional diagrams in absorption



Results from Absorption I

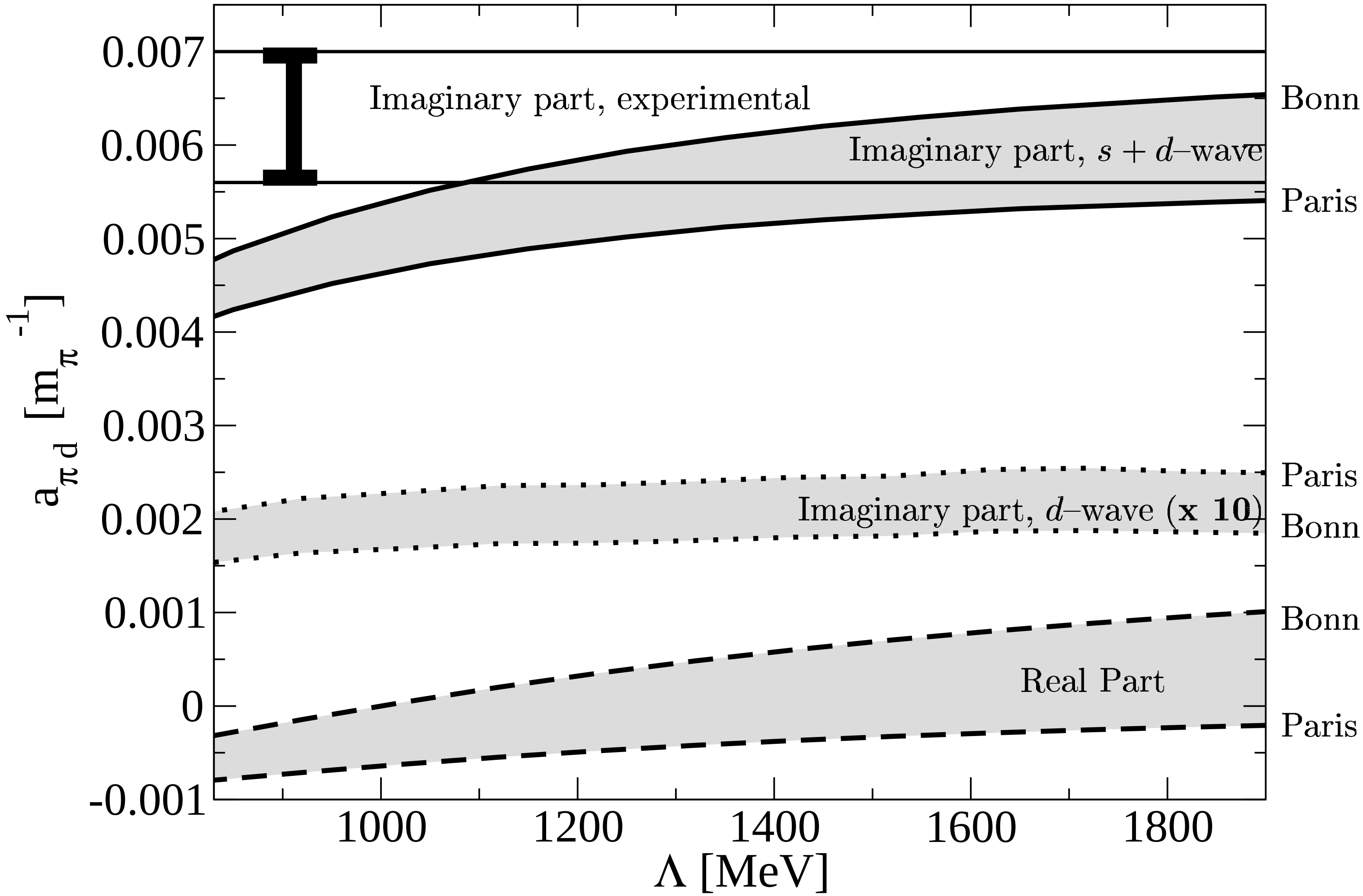
- Real and imaginary contributions from absorption to $a_{\pi^- d}$ for three different approaches. All values in $10^{-4} \cdot m_{\pi^-}^{-1}$:

	(App1)	(App2)	(App3)
Im $a_{\pi^- d}$, s–wave	57.4 ± 5.7	idem	idem
Im $a_{\pi^- d}$, d–wave	2.21 ± 0.33	idem	idem
Im $a_{\pi^- d}$ $s + d$–wave	59.6 ± 5.3	idem	idem
Im $a_{\pi^- d}$ experimental	63 ± 7	idem	idem
Δ Re $a_{\pi^- d}$, s–wave	19.3 ± 8.2	13.6 ± 8.9	2.4 ± 4.3

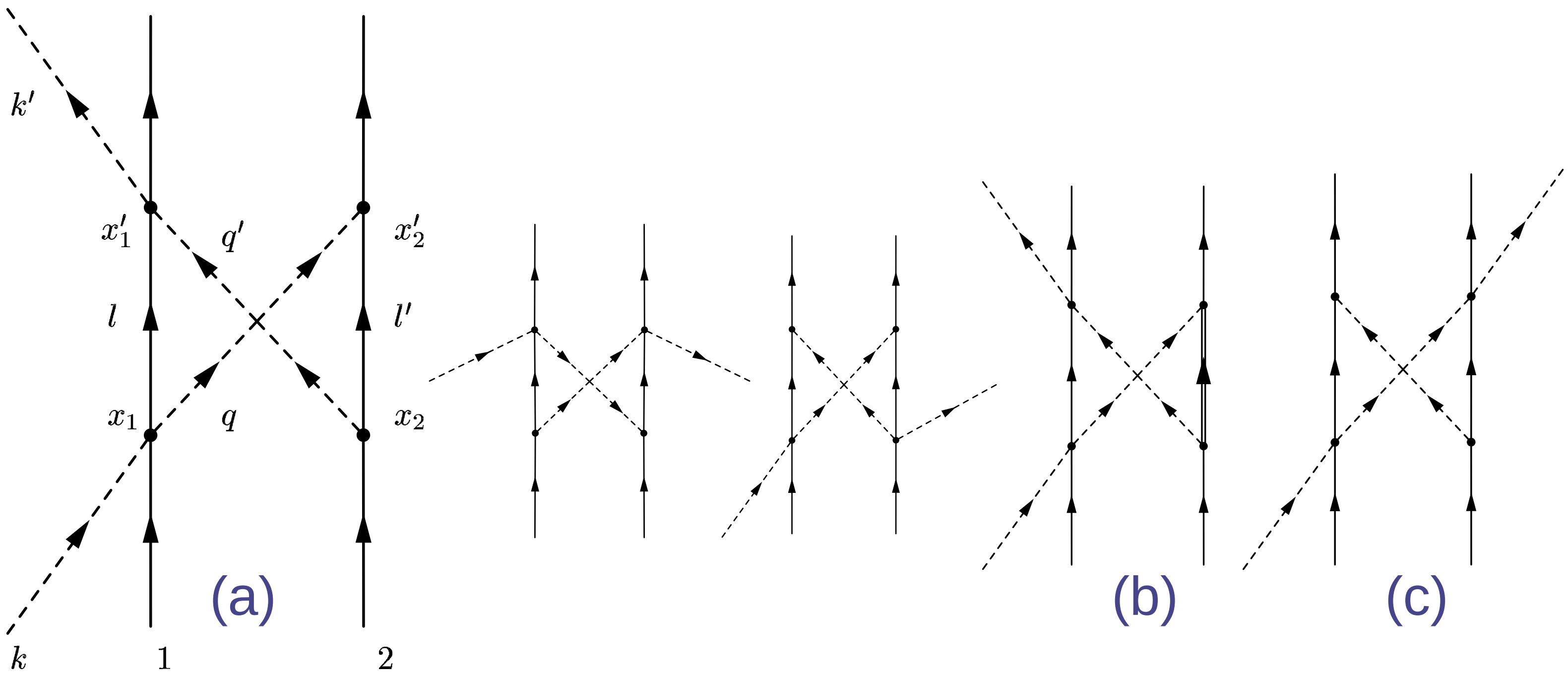
$$\Delta \text{ Re } a_{\pi^- d},\ d - \text{wave, absorption} = 0.18 \cdot 10^{-4} \cdot m_{\pi^-}^{-1}.$$

$$\Delta a_{\pi^- d,\, \mathbf{q}' \times \mathbf{q}} = (0.13 - i\, 0.38) \cdot 10^{-4} \cdot m_{\pi^-}^{-1}$$

Results from Absorption II



Crossed Diagrams



Fermi motion

- p–waves

$$\mathcal{F} = b_0 + b_1(\tilde{\mathbf{t}} \cdot \vec{\tau}) + [c_0 + c_1(\tilde{\mathbf{t}} \cdot \vec{\tau})]\ \mathbf{q}' \cdot \mathbf{q}$$

- Range parameter

$$\begin{aligned} a_{\pi^- p}(\omega) &= a^+ + a^- + \left(b^+ + b^-\right) \mathbf{q}^2 \\ a_{\pi^- n}(\omega) &= a^+ - a^- + \left(b^+ - b^-\right) \mathbf{q}^2 \end{aligned}$$

Single Scattering: $57 \pm 9 \cdot 10^{-4}\ m_\pi^{-1}$

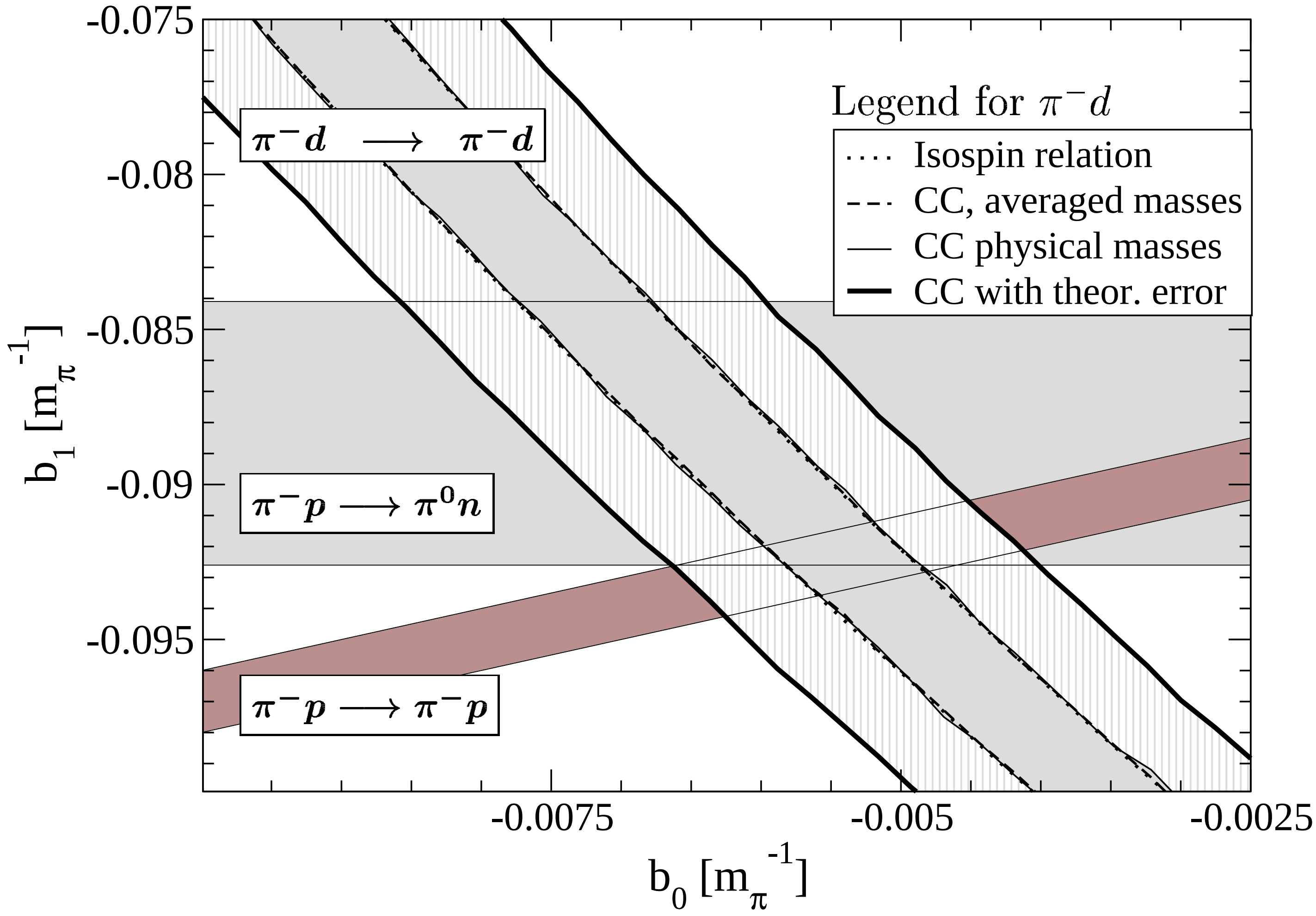
Double Scattering	Bonn		Paris	
$[10^{-4} \cdot m_\pi^{-1}]$	s	d	s	d
leading $s - p$	-30	0	-2	0
subleading $s - p$	13	9	11	13
$p - p$ waves	-3	2	-4	2
Sum	5 ± 15			

Summary of Corrections to $a_{\pi^- d}$

Contribution	Value in $10^{-4} \cdot m_{\pi^-}^{-1}$	Source
$(\pi^- p, \gamma n)$ double scattering	-2	Ericson *et al.*, PRC 66
Form factor/ Non–locality	23 ± 15	Ericson *et al.*, PRC 66 Baru *et al.*, Phys. At. Nucl. 60 Tarasov *et al.*, Phys. At. Nucl. 63
Non–static	11 ± 6	Baru *et al.*, Phys. At. Nucl. 60 Tarasov *et al.*, Phys. At. Nucl. 63 Fäldt, Phys. Sc. 16
virtual pion scattering	-7.1 ± 1.4	Beane, Meissner *et al.*, PRC 57
Dispersion	2.4 ± 4.3	Present study
Crossed π and $\Delta(1232)$	14.6	Present study
Fermi motion, IA	57 ± 9	Present study
Fermi m. double scatt. (s-p,p-p)	5 ± 15	Present study
Sum	104 ± 24	

Constraints on Isoscalar and Isovector

- $a_{\pi^- d} = f(a_{\pi N,i}) + \text{corr.} + (\text{th.+exp. err.}) \simeq g(b_0, b_1)$



Theory I

- Coupled Channel approach with $\pi^- p$, $\pi^0 n$, $\pi^+ p$.
- Lowest order chiral meson baryon Lagrangian Inoue, Oset, Vicente Vacas, NPA 635 :

$$\mathcal{L}^{(1)}_{\pi N} = \text{Tr}\left[\overline{B}\; i\, \gamma^\mu\; \frac{1}{4f^2}\left((\phi\partial_\mu\phi - \partial_\mu\phi\phi)B - B(\phi\partial_\mu\phi - \partial_\mu\phi\phi)\right)\right]$$

- Leads to the kernel

$$V_{ij}\left(\sqrt{s}\right) = -C_{ij}\,\frac{1}{4f_\pi^2}\,\left(2\sqrt{s} - M_i - M_j\right)\sqrt{\frac{M_i + E_i\left(\sqrt{s}\right)}{2M_i}}\sqrt{\frac{M_j + E_j\left(\sqrt{s}\right)}{2M_j}}$$

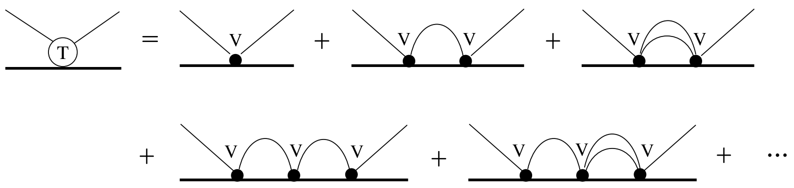
- Isoscalar term from $\mathcal{L}^{(2)}_{\pi N}$ Fettes, Meissner, NPA 679 :

$$V_{ij} \to V_{ij} + \delta_{ij}\left(\frac{4c_1 - 2c_3}{f_\pi^2}\, m_\pi^2 - 2c_2\,\frac{(q^0)^2}{f_\pi^2}\right)\frac{M_i + E_i\left(\sqrt{s}\right)}{2M_i}$$

Theory II

- . . . to be plugged into the Bethe–Salpeter Equation:

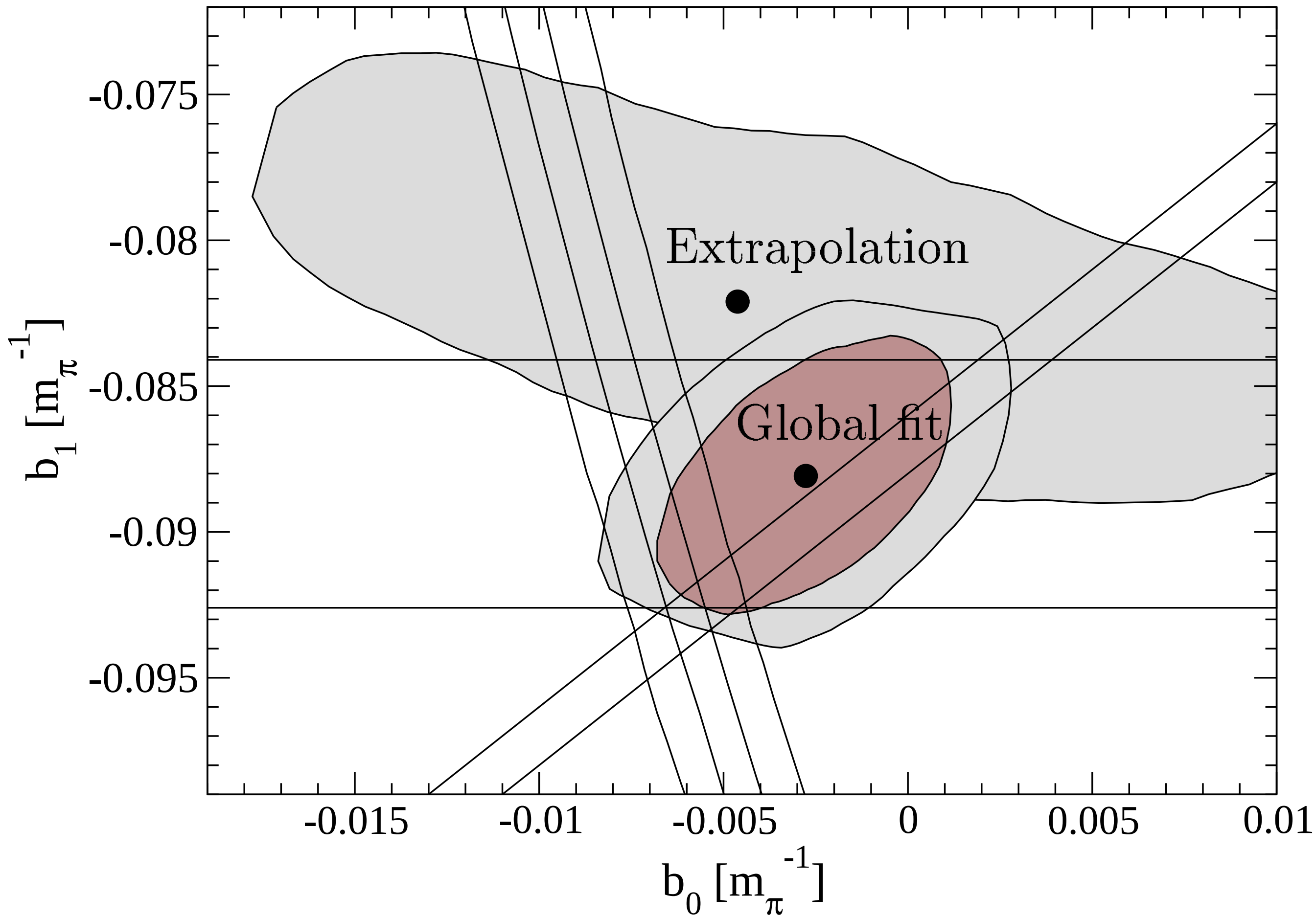
$$T = V + VGT,$$



- Free Parameters:
    - Subtraction constant α from the loops
    - Real part of the $\pi N \to \pi\pi N$ loop
    - 2 from combinations of chiral coefficients c_i from $\mathcal{L}^{(2)}_{\pi N}$
    - Sometimes: Damping factor for quadratic isoscalar, $e^{-\beta^2[(q^0)^2 - m_\pi^2]}$.
    - Sometimes: three different α in $\pi^- p$, $\pi^0 n$, $\pi^+ p$ loops (isospin breaking)
- Fitted Data: $a_{\pi^- p}$, $a_{\pi^- p \to \pi^0 n}$, $a_{\pi^- d}$ (via Faddeev), and πN scattering at finite energies.
- Consistency by Extrapolation, Potential by global fit.

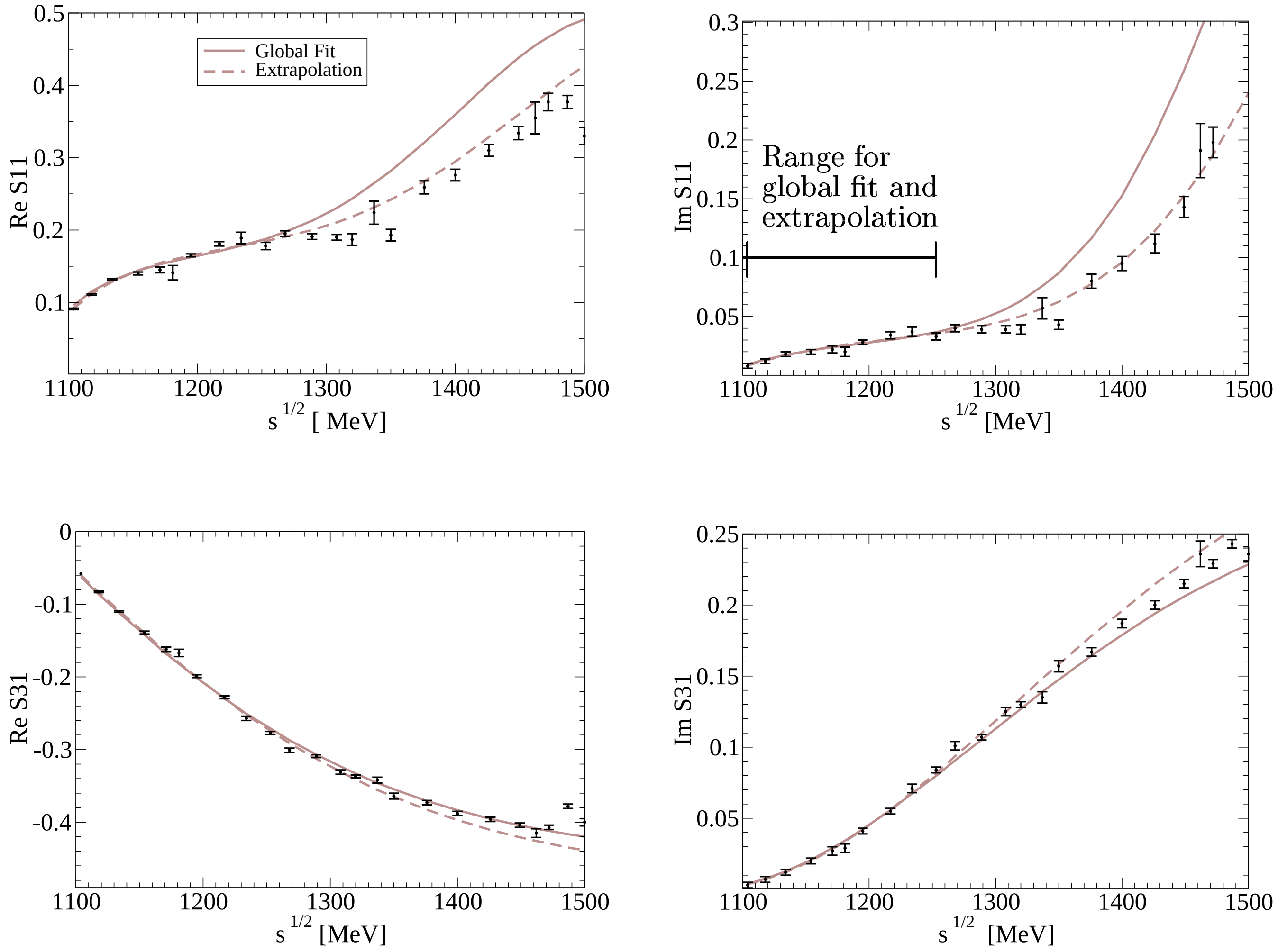
Global fit and Extrapolation

$$(b_0, b_1) = (-28 \pm 40, -881 \pm 48) \cdot [10^{-4} m_\pi^{-1}]$$



Finite Energy Behavior

- Re and Im of s–wave, $I = 1/2$, $I = 3/2$ channels



Data: Arndt *et al.*, nucl-th/0311089, FA02 partial wave analysis

Summary

- πd Scattering is a crucial ingredient in the determination of (b_0, b_1).
- 1–loop corrections: Dispersion, $\Delta(1232)$, crossed diagrams, WFC.
- Im $a_{\pi^- d}$ matches experimental result (cut-off independent).
- Corrections come mainly positive, dispersion is small (literature: larger, negative.
- Large additional attraction from corrections.
- Chiral unitary model: s–wave from $\mathcal{L}_{\pi N}^{(1)}$, $\mathcal{L}_{\pi N}^{(2)}$.
- Inclusion of the $\pi N \to \pi\pi N$ loop.
- Bethe–Salpeter equation gives kernel for πN interaction.
- Fitted: Pionic hydrogen (shift and width), π–deuterium via Faddeev parametrization and low energy πN scattering data.

Conclusions

- Threshold gives quite negative values for (b_0, b_1). From extrapolation: remaining mismatch beween threshold and finite energy πN scattering $\longrightarrow$ uncertainties in phenomenological extraction of partial waves at very low energies ?
- Parametrization of the πN potential found up to $\sqrt{s}$ around 1300 MeV. Good match at energies way beyond the fitted region.
- Chiral coeffients in agreement with χ perturbative calculations with Δ explicit.
- Isospin breaking from mass splitting included in the CC approach. Breaking from this source gives 50% of the one of other studies. Breaking effects from other sources accessible in the model $\longrightarrow$ There is no partial wave analysis free from isospin assumptions.

Parameter Values

	Global Fit	Extrapolation	No damping
fitted data ($\sqrt{s}$)	1104–1253 MeV + threshold	1104–1253 MeV	1104–1180 MeV + threshold
χ^2_r	$51/(2 \cdot 10 + 3) \simeq 2.2$	$24/(2 \cdot 10) = 1.2$	$33/(2 \cdot 6 + 3) = 2.2$
$\alpha_{\pi N}$	-1.143 ± 0.109	-0.990 ± 0.083	-1.528 ± 0.28
$2c_1 - c_3$ [GeV^{-1}]	-1.539 ± 0.20	-1.000 ± 0.463	-0.788 ± 0.14
c_2 [GeV^{-1}]	-2.657 ± 0.22	-2.245 ± 0.45	-1.670 ± 0.07
β [MeV^{-2}]	$0.002741 \pm 1.5 \cdot 10^{-4}$	$0.002513 \pm 3.3 \cdot 10^{-4}$	No β
γ $[10^{-5} \cdot m_\pi^5]$	5.53 ± 7.7	-10 ± 6.1	10 ± 10
$\chi^2(a_{\pi^- p \to \pi^- p})$	3	$[91]$	4
$\chi^2(a_{\pi^- p \to \pi^0 n})$	< 1	$[2]$	< 1
$\chi^2(a_{\pi^- d})$	8	$[6]$	4

- Fettes, Meissner, NPA 679:

$$
\begin{aligned}
2c_1 - c_3 &= -1.63 \pm 0.9 \text{ GeV}^{-1} \\
c_2 &= -1.49 \pm 0.67 \text{ GeV}^{-1}.
\end{aligned}
$$

- Size of the isoscalar:

	Global fit	Extrapolation
b_c $[10^{-4} m_\pi^{-1}]$	-336	-434
b_0 $[10^{-4} m_\pi^{-1}]$, generated	442	396
b_0 $[10^{-4} m_\pi^{-1}]$, final	-28	-46

Parameter Values