Physics of rare kaon and muon decays

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International WS on Nuclear and Particle Physics at 50-GeV PS

December 11, 2001, at KEK

New era of Flavor physics

For a long time, the CP violation was confirmed experimentally only in the $K^0-\overline{K^0}$ mixing. Now, we have observed:

- non-zero ϵ'/ϵ
- CP violation in $B \to J/\psi K_S$ decay.

For a long time, flavor mixing was observed only in the quark sector. Now, we have clear evidences of

- Neutrino oscillation

- CKM Physics and kaon decay:
  $K^+ \to \pi^+ \nu\bar{\nu}$
  $K_L \to \pi^0 \nu\bar{\nu}$
  $K_L \to \pi^0 e^+ e^-$
  $K_L \to \mu^+ \mu^-$

- Forbidden or suppressed decays in the SM:
  T violation in $K^+ \to \pi^0 \mu^+ \nu$
  $K^+ \to \pi^+ e\mu$
  $K_L \to e\mu$

- Muon lepton flavor violation:
  $\mu \to e\gamma$
  $\mu - e$ conversion in muonic atoms
  $\mu \to 3e$
  Muonium - anti-Muonium conversion.

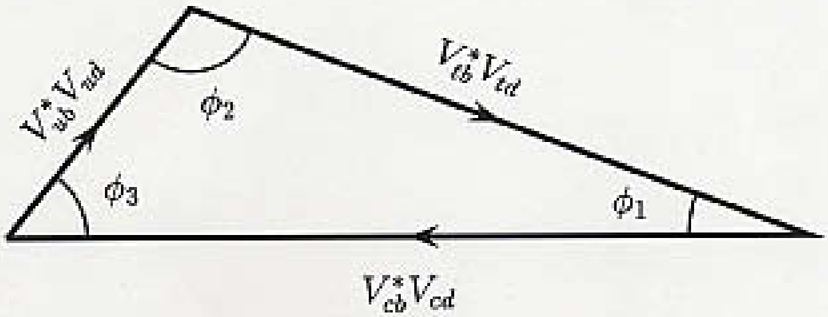
Present status of quark flavor mixing

Consistent determination of the unitarity triangle.

→ Establishing the Kobayashi-Maskawa mechanism of CP violation in the three generation Standard Model.

$$L = -\frac{g}{\sqrt{2}}\overline{u_{iL}}\gamma^{\mu}(V_{CKM})_{ij}d_{jL}W_{\mu} + h.c.$$

$$(V_{CKM})_{ij} \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$



Present constraints on the Unitarity triangle.

- ϵ_K.

- $B_d^0 - \overline{B_d^0}$ mixing. (ΔM_{B_d})

- Charmless b decay.

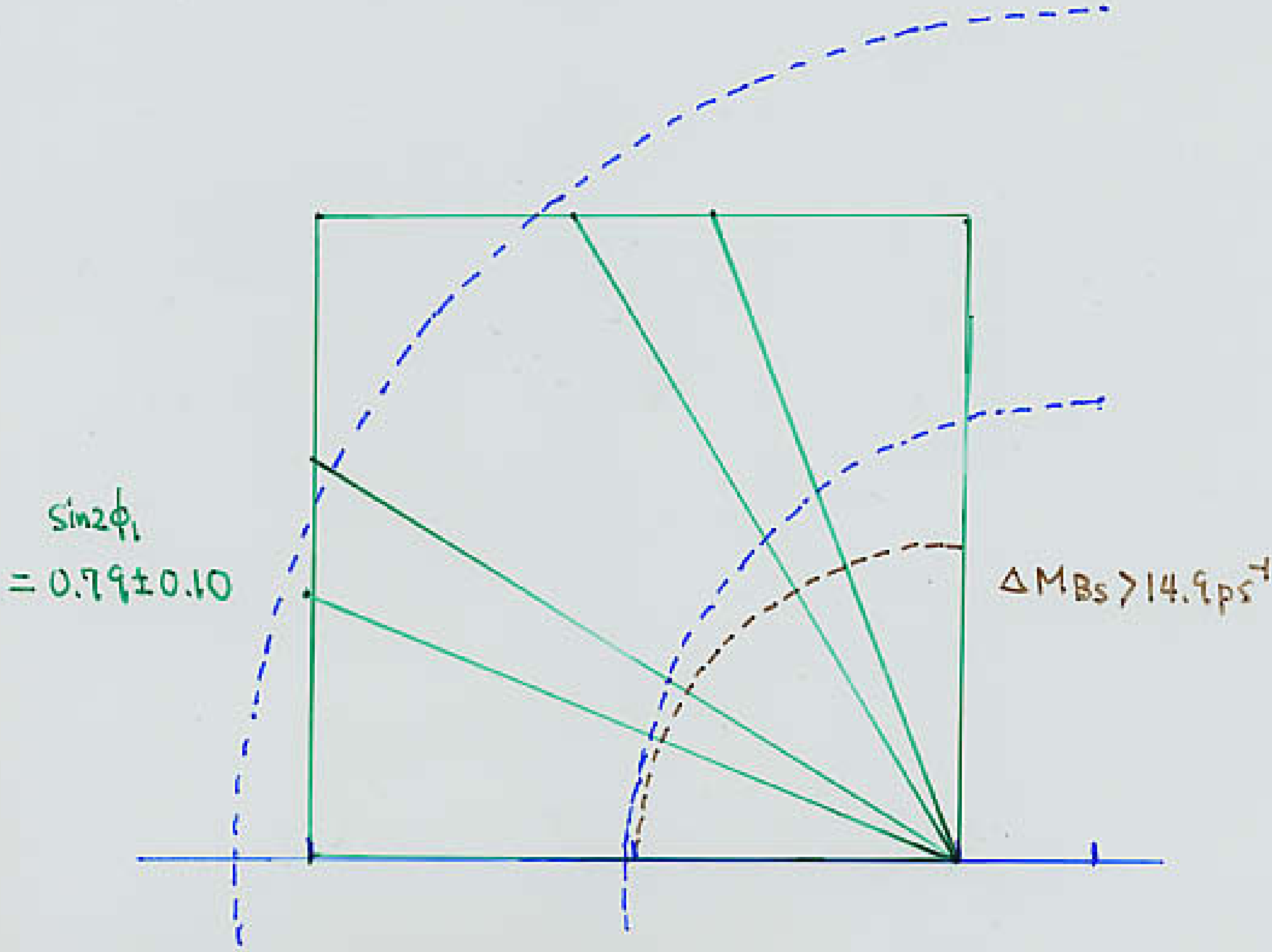
- Time-dependent CP asymmetry in $B \to J/\Psi K_S$ and related modes.

  $$\begin{aligned} \sin(2\phi_1) &= 0.99 \pm 0.14 \pm 0.06 (Belle) \\ & 0.59 \pm 0.14 \pm 0.05 (BaBar) \end{aligned}$$

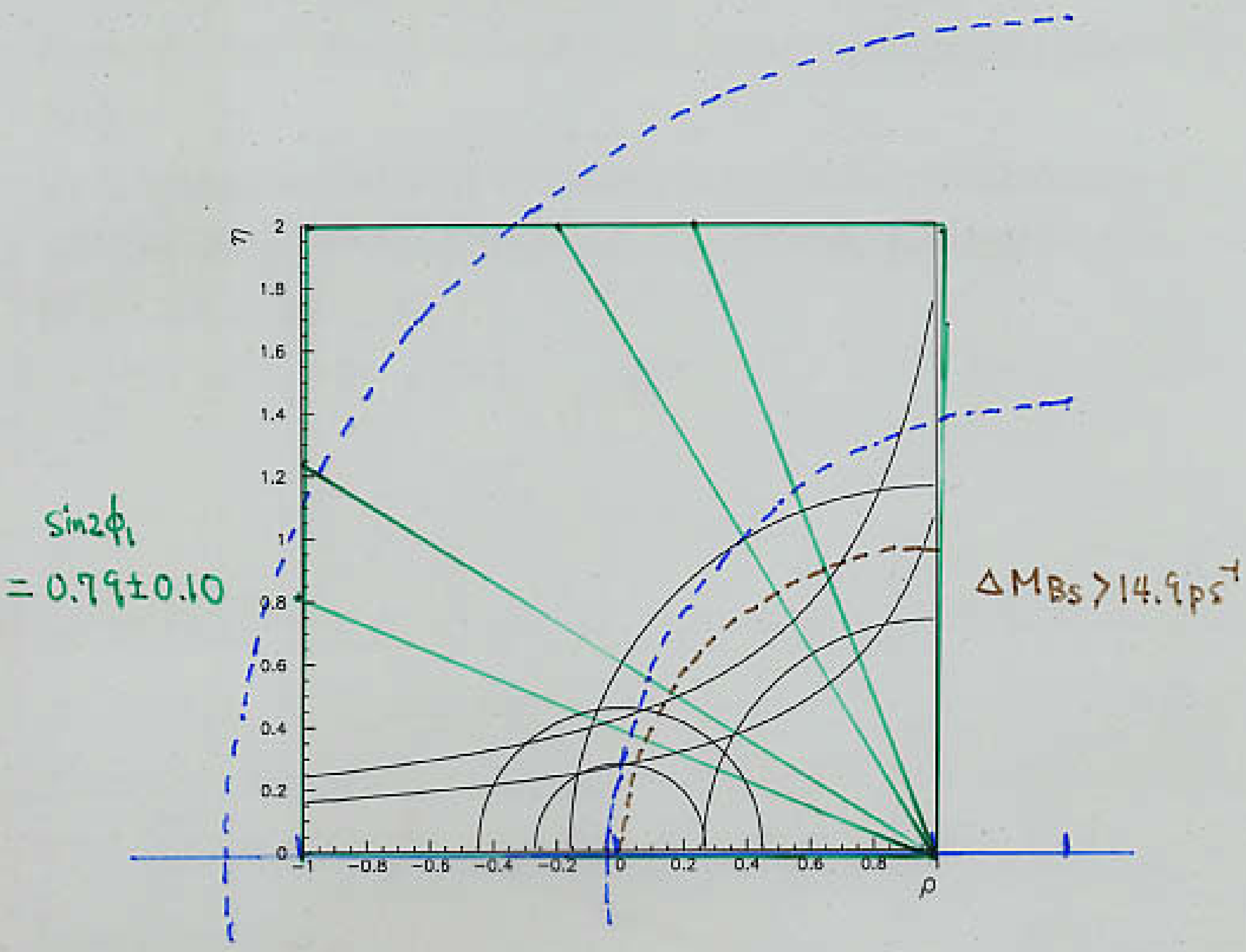
  $\sin(2\phi_1)$ determination is the first crucial test of the KM mechanism of CP violation in the SM.

- Lower bound of ΔM_{B_s}.
  The Tevatron Run-II experiment will determine ΔM_{B_s} in a few years.
  The second crucial test of the KM mechanism.

$B(K^+\to\pi^+\nu\bar{\nu}) = 1.57^{+1.75}_{-0.82}\times10^{-10}$



$B(K^+ \to \pi^+ \nu \bar{\nu}) = 1.57^{+1.75}_{-0.82} \times 10^{-10}$



$K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$

Theoretically very clean processes.

- Little hadronic ambiguity:
  The form factor is determined from $K^+ \to \pi^0 e^+ \nu$.

- QCD corrections are under control.
  Ambiguity from higher orders: $\sim 7\%$ for $K^+ \to \pi^+ \nu\bar{\nu}$ and $O(1)\%$ for $K_L \to \pi^0 \nu\bar{\nu}$.

In the Standard Model,

$$\begin{aligned}
B(K^+ \to \pi^+ \nu\bar{\nu}) &\sim 4.8 \times 10^{-11}(\eta^2 + (1.4 - \rho)^2) \\
&\Leftrightarrow \quad 1.57^{+1.75}_{-0.82} \times 10^{-10} \\
&\qquad (BNL\ E787) \\
B(K_L \to \pi^0 \nu\bar{\nu}) &\sim 2.0 \times 10^{-10}\eta^2 \\
&\Leftrightarrow \quad < 5.9 \times 10^{-7}\ (90\% CL) \\
&\qquad (KTeV)
\end{aligned}$$

If new physics effects dominate in $K_L \to \pi^0 \nu\bar{\nu}$, there are two possibilities. Y.Grossman and Y.Nir 1997

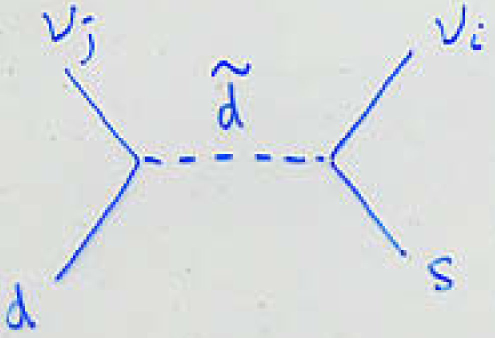
Without lepton flavor violation, a model-independent bound follows from $B(K_L \to \pi^+ \nu\bar{\nu})$

$$B(K_L \to \pi^0 \nu\bar{\nu}) < 4.4 B(K^{+}_{\cancel{L}} \to \pi^+ \nu\bar{\nu}).$$

If some lepton flavor violating interaction exists,

$$B(K_L \to \pi^0 \nu_i \bar{\nu}_j)$$

can occur through CP conserving interactions. (ex. R parity violating SUSY model)



Future flavor physics

In 2006, the LHC experiment will start. Physics issues will depend on outcomes of the LHC experiment. (SUSY may be discovered, or disproved.)

"New physics search" will be a major interest for flavor physics. For this purpose, it is important to look for new physics effects in several theoretically clean processes in K and B decays, because pattern of the deviation from the SM provides us with a hint for new physics.

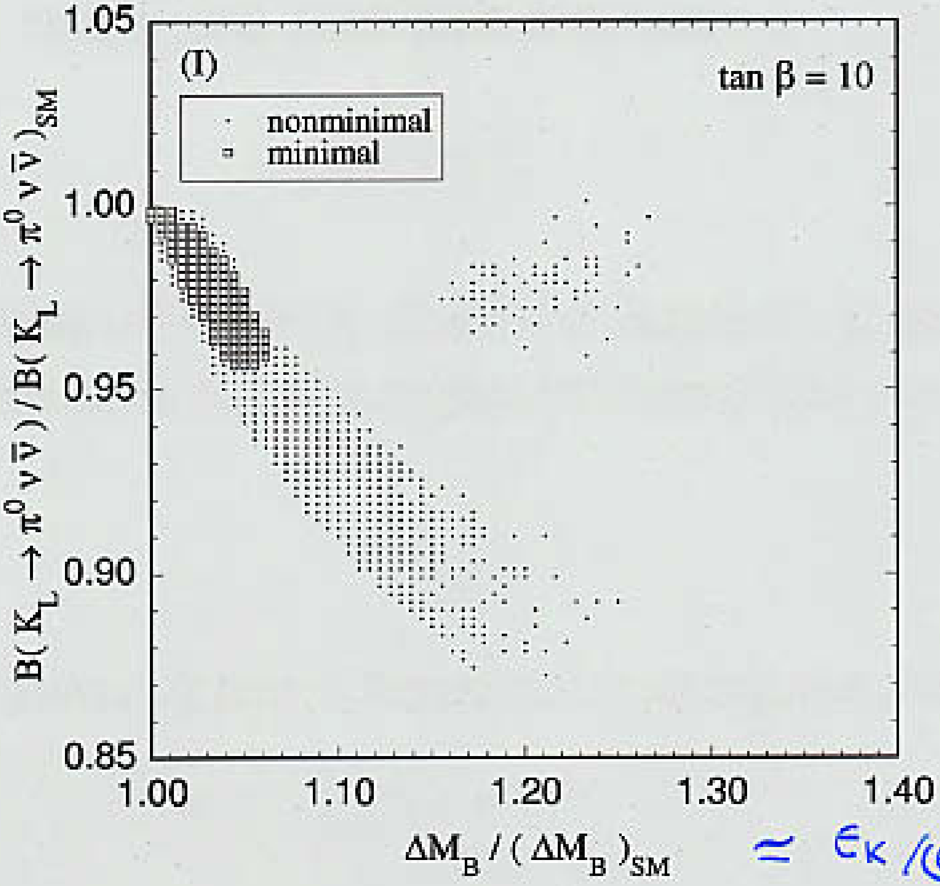
Example: Minimal Supergravity model.

SUSY effects:

- enhance ϵ_K and ΔM_{B_d},
- suppress $B(K \to \pi\nu\bar{\nu})$,
- do not change $\Delta M_{B_s}/\Delta M_{B_d}$ and $A_{CP}(B \to J/\Psi K_s) etc.$.

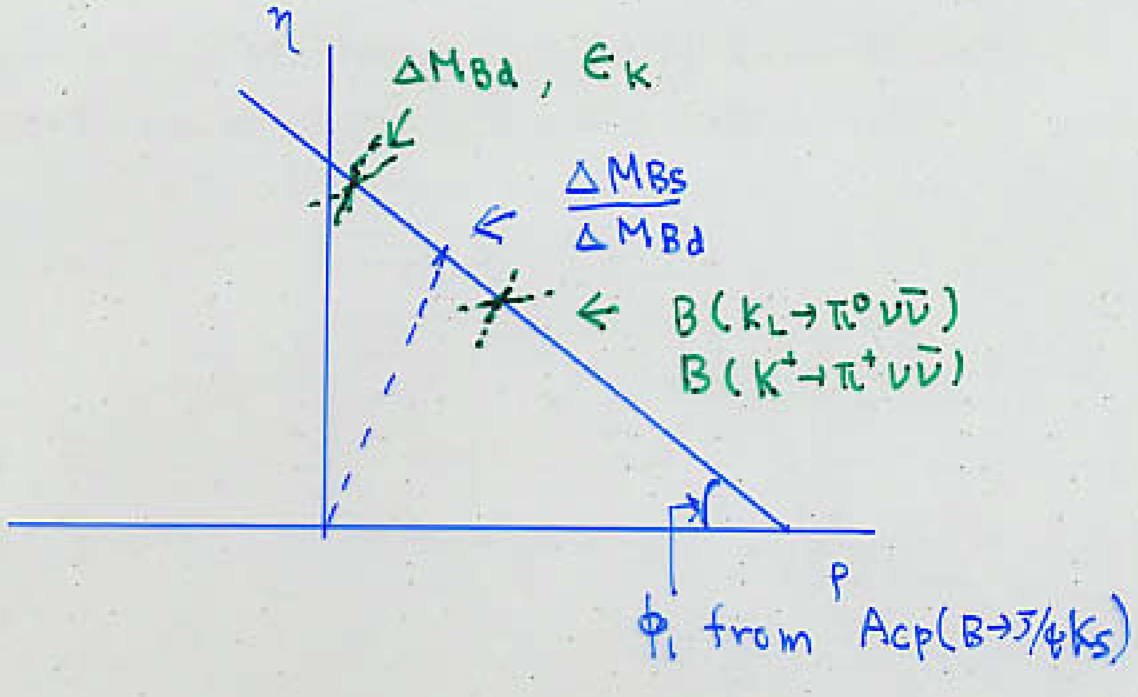
Minimal Supergravity Model

T.Goto, Y.Okada and Y.Shimizu, 1999

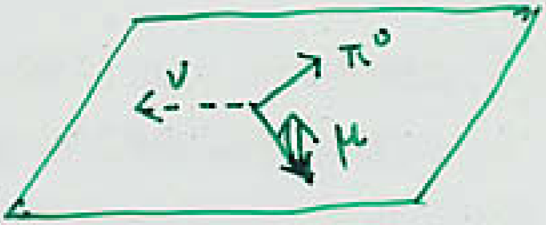


$B(K^+ \to \pi^+ \nu\bar{\nu}) / B(K^+ \to \pi^+ \nu\bar{\nu})_{SM} \simeq$

$\simeq \epsilon_K / (\epsilon_K)_{SM}$



T violation in $K^+ \to \pi^0\mu^+\nu$
Transverse μ^+ polarization.



$$\vec{\sigma}\cdot(\vec{p_\pi}\times\vec{p_\mu})$$

↔ T-odd quantity.

$$P_T(K^+ \to \pi\mu^+\nu) = (-3.1 \pm 3.7 \pm 0.9)\times 10^{-3}$$

(KEK E246)

- Small T violation effect from the KM phase in the SM ($O(10^{-7})$).
- Small fake T violation from final state interaction ($O(10^{-6})$). A.R.Zhitnitskii, 1980, V.P.Efrosinin, I.B.Khriplovich, G.G.Kirilin and Yu.G.Kudenko, 2000

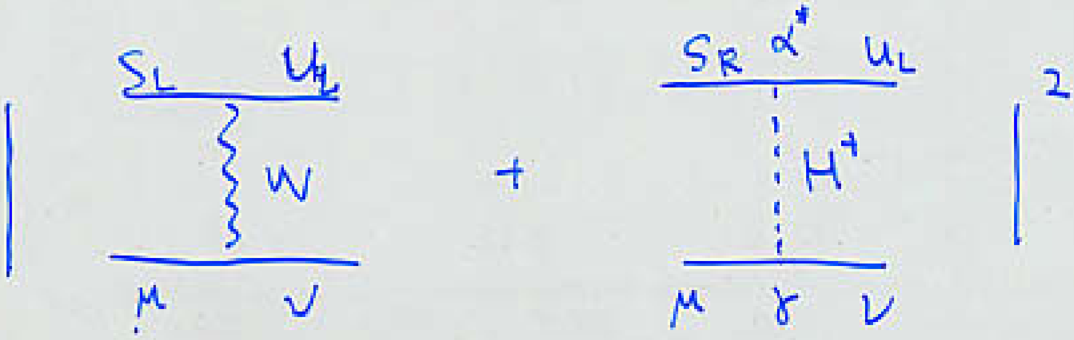
An important process to search for new source of CP violation.

- Multi-Higgs-doublet model ($n \geq 3$).
- SUSY model with general flavor mixing.

Multi-Higgs doublet model ($n \geq 3$).

$$\mathcal{L} = (2\sqrt{2}G_F)^{\frac{1}{2}} \sum_{i=1}^{n-1} \{\alpha_i \bar{u}_L V_{CKM} M_D d_R H_i^+$$
$$+ \beta_i \bar{u_R} M_U V_{CKM} d_L H_i^+ + \gamma_i \bar{\nu}_L M_E e_R H_i^+\} + h.c.$$

T odd effects arise as an interference term between the $W^\pm$ and the $H^\pm$ exchange diagrams.



$$P_T(K^+ \to \pi\mu^+\nu) \sim -0.2\frac{m_K^2}{m_H^2} Im(\gamma_1\alpha_1^*)$$

Using the bound on $|Im(\gamma_1\alpha_1^*)|$ from $B \to \tau\nu X$,

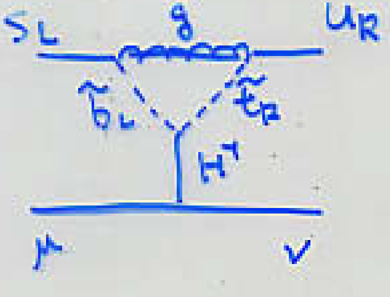
$$|P_T(K^+ \to \pi\mu^+\nu)| \lesssim 1 \times 10^{-2}$$

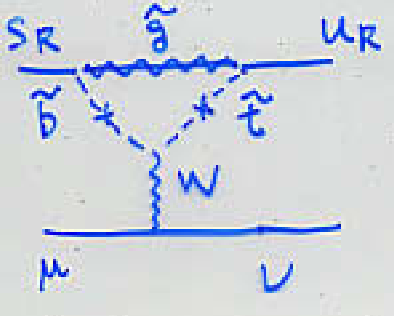
$P_T(K^+ \to \pi^0 \mu^+ \nu)$ v.s. $P_T(K^+ \to \gamma \mu^+ \nu)$

Transverse μ^+ polarization in $K^+ \to \gamma\mu^+\nu$ also implies T violation.

Correlation between two quantities depends on models beyond the SM.

- Multi-Higgs doublet model.
  M.Kobayashi, T.T.Lin and Y.Okada 1996
  $$P_T(\pi^0\mu^+\nu) \sim 2P_T(\gamma\mu^+\nu).$$
- SUSY model with large squark flavor mixing. G.H.Wu and J.N. Ng 1996
  $$P_T(\pi^0\mu^+\nu) \sim -2P_T(\gamma\mu^+\nu),$$
  for a loop induced $s_L - \bar{u_R} - H^+$ coupling.
  $$P_T(\pi^0\mu^+\nu) \sim 0,\ P_T(\gamma\mu^+\nu) \neq 0,$$
  for a loop induced $s_R - \bar{u_R} - W^+$ coupling.





Lepton Flavor Violation (LFV) in charged lepton processes

$\Delta L_f = 1$

$\mu^+ \to e^+\gamma$

$\mu^+ \to e^+e^+e^-$

$\mu^- - e^-$ conversion in muonic atoms

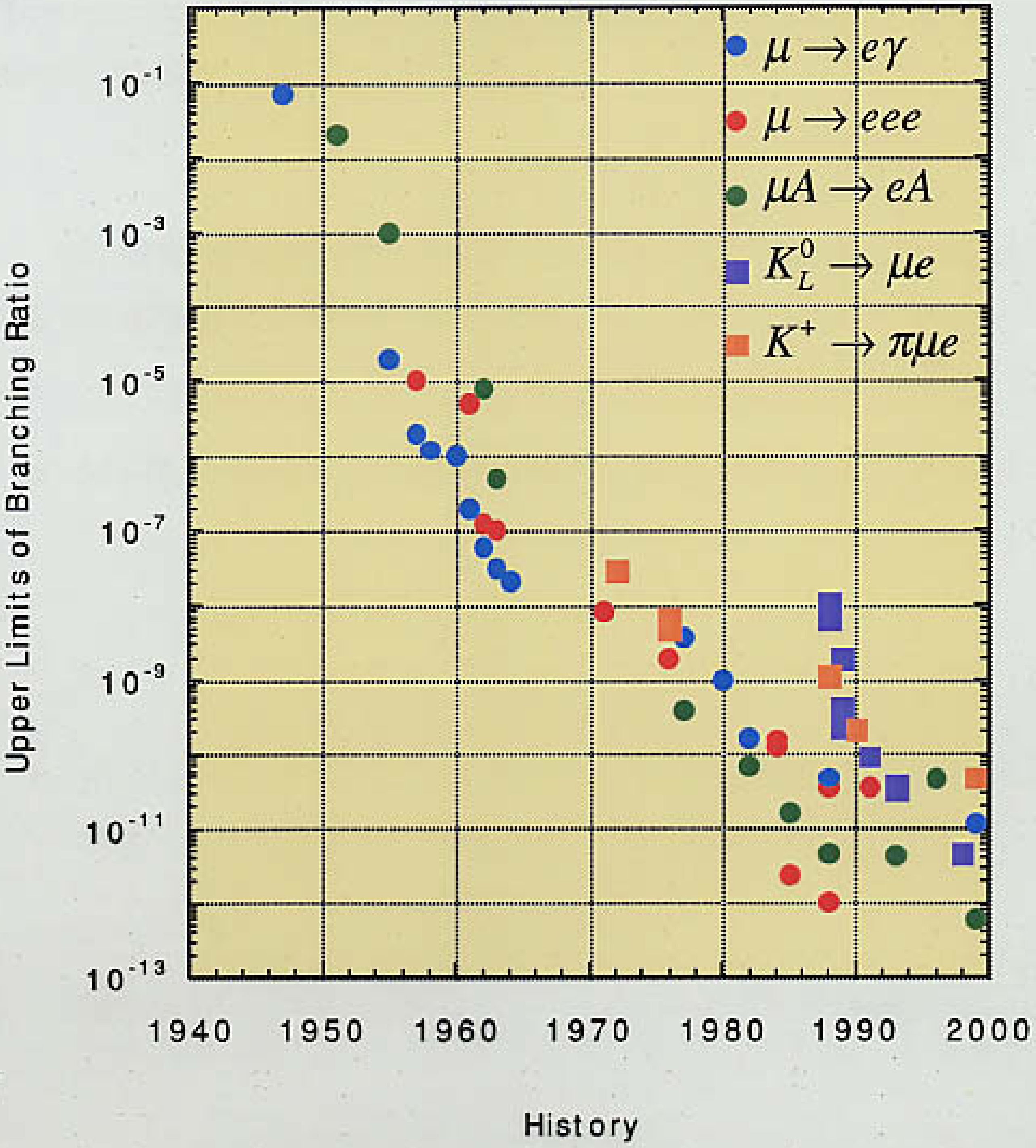
$\tau \to \mu\gamma$

$\tau \to 3\mu$

$\Delta L_f = 2$

Muonium (μ^+e^-) - Anti-Muonium (μ^-e^+) conversion

The Minimal Standard Model $\Longrightarrow$ No LFV
The simplest extension of SM with Sea-Saw or Dirac neutrino
$\Longrightarrow$ Charged-lepton LFV is negligibly small
($B(\mu \to e\gamma) < 10^{-40}$)
A clear signal of physics beyond the SM (with neutrino oscillation)


$\mu \rightarrow e\gamma$
$\mu \rightarrow eee$
$\mu A \rightarrow eA$
$K_L^0 \rightarrow \mu e$
$K^+ \rightarrow \pi\mu e$
Upper Limits of Branching Ratio
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
1940
1950
1960
1970
1980
1990
2000
History

Experimental Bounds

Process	Current	Future
B($\mu^+ \to e^+\gamma$)	1.2×10^{-11}	10^{-14}(PSI)
B($\mu^+ \to e^+e^+e^-$)	1.0×10^{-12}	
$\frac{\sigma(\mu^- T_i \to e^- T_i)}{\sigma(\mu^- T_i \to capture)}$	6.1×10^{-13}	
$\frac{\sigma(\mu^- A_l \to e^- A_l)}{\sigma(\mu^- A_l \to capture)}$		10^{-16}(MECO)
B($\tau \to \mu\gamma$)	1.1×10^{-6}	
B($\tau \to 3\mu$)	1.9×10^{-6}	
$G_{Mu\overline{Mu}}/G_F$	3×10^{-3}	

$(H_{Mu\overline{Mu}} = \frac{G_{Mu\overline{Mu}}}{\sqrt{2}}\overline{\mu}\gamma_\lambda(1-\gamma_5)e\overline{\mu}\gamma^\lambda(1-\gamma_5)e + H.c.)$

Examples of New Physics

$\Delta L_f = 1$

SUSY (SUSY GUT, SUSY model with right-handed neutrino)

R-parity violating SUSY model

Model with extra-dimension

Model with violation of Lorentz invariance

……

$\Delta L_f = 2$

R-parity violating SUSY model

Left-Right Model with triplet Higgs field

Model with bilepton

……

Comparison of three processes.

For $\mu^+ \to e^+\gamma$:

$$\begin{aligned} L_{photon} &= m_\mu A_R \overline{\mu_R} \sigma^{\mu\nu} e_L F_{\mu\nu} \\ &+ m_\mu A_L \overline{\mu_L} \sigma^{\mu\nu} e_R F_{\mu\nu} + h.c. \end{aligned}$$

For $\mu^+ \to e^+e^+e^-$:

$$L_{photon} + \bar{\mu}e\bar{e}e \ four-fermion \ int.$$

For $\mu^- - e^-$ conversion in muonic atoms:

$$L_{photon} + \bar{e}\mu\bar{q}q \ four-fermion \ int.$$

Relationship among three branching ratios depends on theoretical models. In many cases in SUSY GUT and SUSY with right-handed neutrino, L_{photon} gives dominate contributions for all three. In such a case,

$$\begin{aligned} B(\mu^+ \to e^+e^+e^-) &\sim 6 \times 10^{-3} B(\mu \to e\gamma) \\ \frac{\sigma(\mu^- T_i \to e^- T_i)}{\sigma(\mu^- T_i \to capture)} &\sim 4 \times 10^{-3} B(\mu \to e\gamma) \end{aligned}$$

Other possibilities can be realized in some parameter space of the SU(5) GUT and SUSY models without R parity conservation.

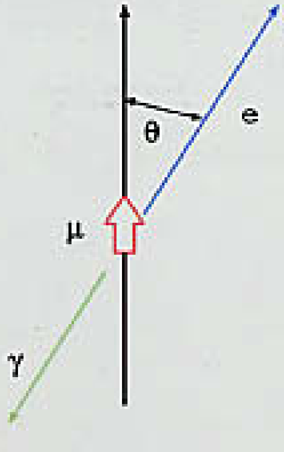
→ Discrimination of models

Polarized muon and LFV search

Highly polarized muon is available for rare muon experiments. μ^+ from π^+ stopped at the surface of the targets is 100 % polarized. (Surface muon)

- The polarized muons are useful to suppress the background processes for $\mu^+ \to e^+\gamma$ search. Y.Kuno and Y.Okada 1996; Y.Kuno, A.Maki and Y.Okada 1997

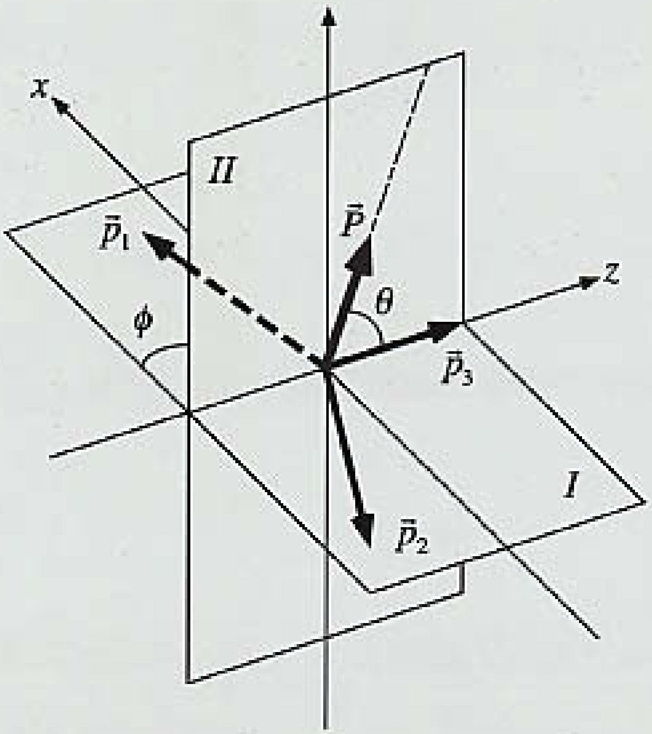
- For $\mu^+ \to e^+\gamma$, the chirality of interaction can be determined by angular distribution.



$$\frac{dB(\mu^+ \to e^+\gamma)}{d\cos\theta} \propto 1 + A_{\mu\to e\gamma} P_\mu \cos\theta$$

where $(A_{\mu\to e\gamma} \equiv \frac{|A_L|^2-|A_R|^2}{|A_L|^2+|A_R|^2})$

- For $\mu^+ \to e^+e^+e^-$, T-odd and P-odd asymmetries can be defined.



$$\begin{aligned} A_{P_1} &= \frac{N(P_z>0)-N(P_z<0)}{N(P_z>0)+N(P_z<0)} \\ A_{P_2} &= \frac{N(P_x>0)-N(P_x<0)}{N(P_x>0)+N(P_x<0)} \\ A_T &= \frac{N(P_y>0)-N(P_y<0)}{N(P_y>0)+N(P_y<0)} \end{aligned}$$

SU (5)& SO (10) SUSY GUT models

	SU (5)	SO (10)
$A_{\mu\to e\gamma}$	+100%	-100% – +100%
A_{P_1}	-30% – +40 %	$\simeq -A_{\mu\to e\gamma}/10$
A_{P_2}	-20% – +20 %	$\simeq -A_{\mu\to e\gamma}/6$
$\|A_T\|$	$\lesssim$15%	$\lesssim$ 0.01%

Y.Okada, K.Okumura and Y.Shimizu, 2000

Z-dependence of the $\mu - e$ conversion rate

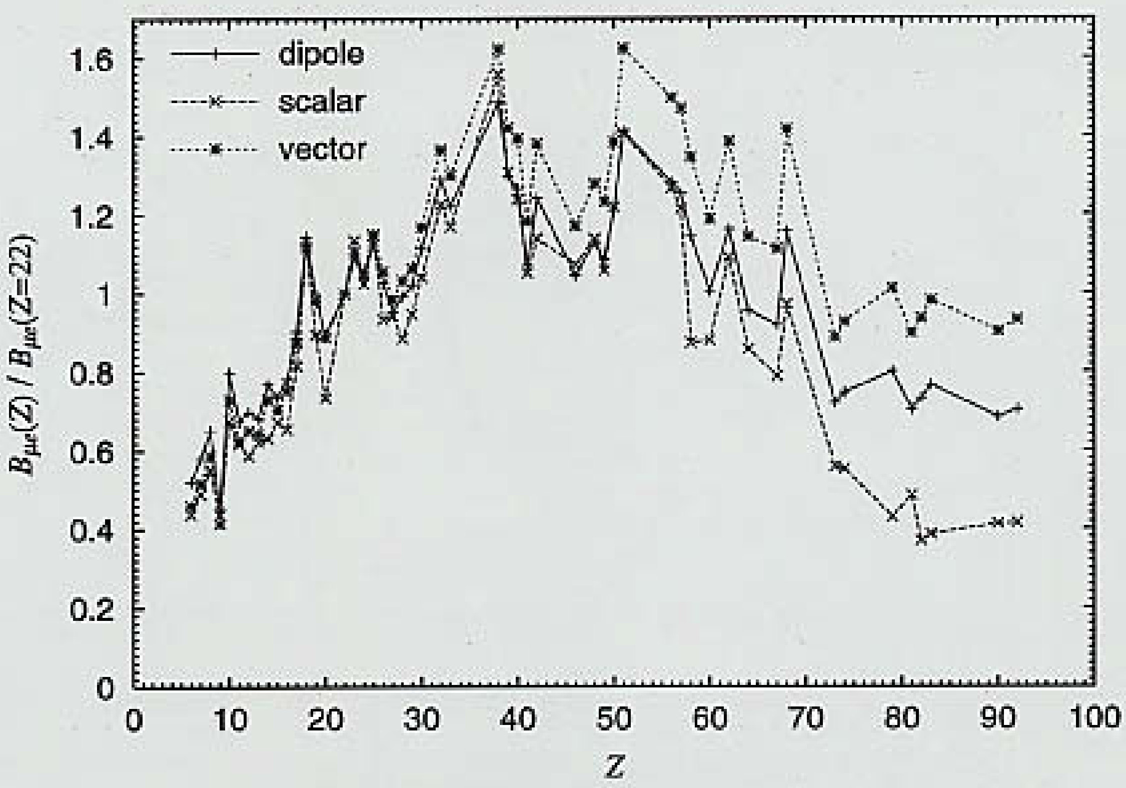
$$B_{\mu e} \equiv \frac{\sigma(\mu^- N(A,Z) \to e^- N(A,Z))}{\sigma(\mu^- N(A,Z) \to capture)}$$

$$\sigma(\mu^- N(A,Z) \to e^- N(A,Z)) \\ \sim \int d^3x (\overline{\psi^{(e)}} \psi_{1s}^{(\mu)} \rho^{(p,n)})$$

- Since the conversion process is coherent process, a nucleus with a larger Z is preferred, but for a large Z, there is a suppression by form factors.

- For a heavy nucleus, relativistic effects and Coulomb distortion of the electron wave function become important.

  S.Weinberg and G.Feinberg, 1959; O.Shanker, 1979; A.Czarnecki, W.J.Marciano and K.Melnikov, 1997



R.Kitano, M.Koike and Y.Okada, 2001

Different Z-dependences for various types of interactions.

→ Discrimination of models

Summary

- Both B decay and $K \to \pi\nu\bar{\nu}$ experiments are important for precise determination of the CKM parameters and search for physics beyond the SM through the unitarity triangle.

- Other K decay experiments such as the T violation search in $K^+ \to \pi^0\mu^+\nu$ provide us direct ways to look for new physics.

- Search for lepton flavor violation is very important because well-motivated models beyond the SM predict large branching ratios.

- All three processes, $\mu^+ \to e^+\gamma$, $\mu^+ \to e^+e^+e^-$ and $\mu^- - e^-$ conversion are important to distinguish various models. Polarized muon experiments are also useful.